Manonmaniam Sundaranar University,

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OPEN AND DISTANCE LEARNING (ODL) PROGRAMMES

(FOR THOSE WHO JOINED THE PROGRAMMES FROM THE ACADEMIC YEAR 2023–2024)

B.Sc. Physics

III Year

Electricity, Magnetism and Electromagnetism

Course Material

Prepared

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Electricity, Magnetism and Electromagnetism

Unit 1

CAPACITORS AND THERMO ELECTRICITY

Capacitor -principle -capacitance of a parallel plate capacitor (with and without dielectric slab)-effect of dielectric – Carey Foster bridge – temperature coefficient of resistance – Seebeck effect -Laws of thermo emf - Peltier effect -Thomson effect-Thermoelectric diagrams and their uses -thermodynamics of thermocouple.

Unit 2

MAGNETIC EFFECT OF CURRENT

Biot and Savart‘s law-magnetic induction due to circular coil –force on a current element by magnetic field -force between two infinitely long conductors -torque on a current loop in a field -moving coil galvanometer -damping correction Ampere ‘s circuital law -differential form –divergence of magnetic field magnetic induction due to toroid.

Unit 3

MAGNETISM AND ELECTROMAGNETIC INDUCTION

Magnetic induction B -Magnetization M -relation between B, H and M magnetic susceptibility-magnetic permeability-experiment to draw B-H curve energy loss due to hysteresis -importance of hysteresis curve –Faraday and Lenz laws -vector form -self-inductance -coefficient of self-inductance of solenoid Anderson ‘s method -mutual inductance -coefficient of mutual inductance between two coaxial solenoids -coefficient of coupling.

Unit 4

TRANSIENT AND ALTERNATING CURRENTS

Growth and decay of current in a circuit containing resistance and inductance growth and decay of charge in a circuit containing resistance and capacitor growth and decay of charge in an LCR circuit (expression for charge only)-peak, average and rms values of ac-LCR series-parallel circuits –resonance condition - Q factor -power factor.

Unit 5
MAXWELL'S EQUATIONS AND ELECTROMAGNETIC WAVES

Maxwell's equations in vacuum, material media-physical significance of Maxwell s equations – displacement current-plane electromagnetic waves in free space velocity of light, Poynting vector –electromagnetic waves in a linear homogeneous media -refractive index.

TEXT BOOKS

1. Murugeshan. R., Electricity and Magnetism, 8^{th} Edn, 2006, S. Chand and Co, New Delhi.

2. Sehgal D. L., Chopra K. L, Sehgal N.K., -Electricity and Magnetism, Sultan Chand and Sons, New Delhi.

3. M. Narayanamurthy and N. Nagarathnam, Electricity and Magnetism, 4th Edition. National Publishing Co., Meerut.

Unit 1: Capacitors and Thermo Electricity

1. Introduction
2. Capacitance of a parallel plate capacitor
3. Effect of Dielectric
4. Carey Foster Bridge
5. Temperature of Coefficient of Resistance
6. Seebeck Effect
7. Laws of Thermo emf
8. Peltier effect
9. Thomson Effect
10. Thermoelectric Diagrams and their uses
11. Thermodynamics of Thermocouple.

1.1 Introduction

A capacitor is a fundamental passive component used in electrical and electronic circuits for the purpose of storing electrical energy in the form of an electric field. It consists essentially of two conducting surfaces (called plates) separated by an insulating medium known as the dielectric. When a potential difference is applied across the plates, electric charges accumulate on them: one plate becomes positively charged and the other negatively charged. The dielectric between the plates prevents direct conduction, allowing the device to store energy.

The property of a capacitor to store electrical charge is termed **capacitance**. Capacitance depends on three primary factors:

(1) the surface area of the plates,

(2) the distance between them, and

(3) the permittivity of the dielectric material used.

Mathematically, capacitance is defined as the ratio of the charge stored on one plate to the potential difference between the plates. The SI unit of capacitance is the **farad (F)**. Capacitors are widely employed in various applications, such as filtering, coupling and decoupling signals, timing circuits, energy storage, power factor correction, and smoothing outputs in power supplies. They play a crucial role in AC circuits by allowing alternating current to pass while blocking direct current, making them vital elements in electronic communication systems.

Thermoelectricity refers to the class of phenomena that involve direct conversion between thermal energy and electrical energy. These effects arise due to the movement of charge carriers (electrons or holes) in a material when subjected to temperature differences. Thermoelectric phenomena form an important part of solid-state physics and have significant applications in temperature measurement, cooling systems, and power generation.

The field is based on three fundamental effects:

1. **Seebeck Effect** – When two dissimilar conductors are joined to form a closed loop and the junctions are maintained at different temperatures, an electromotive force is generated in the circuit. This effect forms the basis of thermocouples and thermoelectric power generation.
2. **Peltier Effect** – When an electric current is passed through a junction of two different materials, heat is either absorbed or evolved at the junction depending on the direction of the current. This effect is used in thermoelectric cooling devices.

3. **Thomson Effect** – A conductor carrying current and having a temperature gradient either absorbs or releases heat continuously along its length. This effect is observed in uniform conductors and complements the Seebeck and Peltier effects.

Thermoelectric materials and devices are valued for their solid-state operation, absence of moving parts, high reliability, and ability to function in extreme environments. Applications include thermoelectric generators (TEGs), thermoelectric coolers (TECs), spacecraft power systems, industrial waste heat recovery, and precision temperature control.

1.2 Capacitance of a parallel plate capacitor

A **parallel-plate capacitor** consists of two large, flat, conducting plates of area Aplaced parallel to each other at a small separation d. When the plates carry equal and opposite charges $+Q$and $-Q$, an electric field is established between them and electrical energy is stored in the field. In the absence of any dielectric material between the plates (i.e., plates separated by vacuum or air), the capacitance depends only on geometry and the permittivity of free space.

Capacitance $C$is defined as the ratio of charge on one plate to the potential difference between the plates:

$$C = \frac{Q}{V}$$

where $Q$is the magnitude of charge on each plate and $V$is the potential difference between plates.

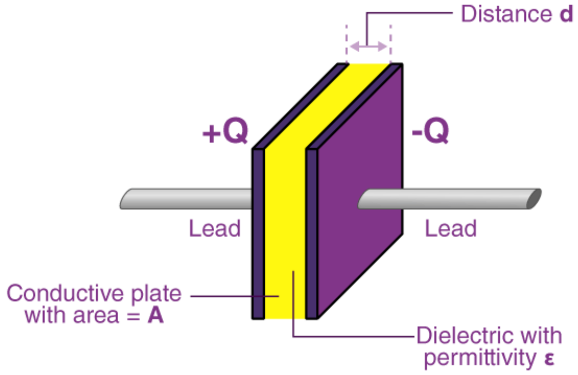


Figure 1. 1 (Parallel Plate Capacitor)

Assume plates are large and separation $d$is small compared to plate dimensions, so edge effects (fringing) are negligible. For an infinite sheet of surface charge density $\sigma$the field magnitude on one side is $E = \frac{\sigma}{2\varepsilon_0}$. Two oppositely charged parallel plates produce a uniform field between them:

- Surface charge density on plate: $\sigma = \frac{Q}{A}$.
- Net field between the plates (directed from + to −):

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

(Outside the plates the fields cancel, ideally $E = 0$.)

Here $\varepsilon_0$is the permittivity of free space, $\varepsilon_0 \approx 8.854 \times 10^{-12}\ \mathrm{F\ m^{-1}}$.

Potential difference Vbetween plates separated by distance $d$in a uniform field is

$$V = E\,d = \frac{Q}{\varepsilon_0 A}\,d.$$

Using $C = Q/V$:

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q\,d}{\varepsilon_0 A}} = \frac{\varepsilon_0 A}{d}.$$

Thus for a parallel-plate capacitor in vacuum (or air, approximately),

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

Units: farads (F). Practical units are much smaller (μF, nF, pF) because $\varepsilon_0$is small.

Energy Ustored in a capacitor:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV.$$

For the parallel-plate capacitor,

$$U = \frac{1}{2} \frac{\varepsilon_0 A}{d} V^2.$$

Limitations of the ideal formula

1. **Fringing fields:** For finite plates, field lines bulge near edges; the simple formula underestimates actual capacitance slightly. Fringing becomes important when $d$is not very small compared to the linear dimensions of the plates.
2. **Surface roughness and finite plate thickness** alter effective area.
3. **Air as dielectric:** Formula uses ε_0. For air $\varepsilon \approx \varepsilon_0(1+\chi)$where $\chi$is small; using $\varepsilon_0$is typically acceptable unless high precision required.
4. **Breakdown voltage:** Maximum usable $V$is limited by dielectric breakdown of air (spark/ionization across gap).

Objective

To determine capacitance of a parallel-plate capacitor and verify the relation $C = \frac{\varepsilon_0 A}{d}$.

Apparatus

- Two conducting plates (rectangular or circular) with known area A.
- Nonconducting adjustable spacer or micrometer to set plate separation d.
- DC power supply (low voltage), or electrostatic voltmeter/charging source.
- Sensitive electrometer or digital capacitance meter (preferred), or an arrangement with known resistor and timing (RC method).
- Connecting wires, insulating stand, ruler/micrometer, vernier caliper.
- Switch, stopwatch (if using RC time constant method).

Methods

Method A — Direct capacitance meter / LCR meter (recommended)

1. Mount plates parallel on insulating supports; measure plate area A. Ensure plates are clean and parallel.
2. Set separation $d$with micrometer; measure d.
3. Connect plates to a capacitance meter and record measured capacitance C_{exp}.
4. Repeat for different d(e.g., several values) and record C_{exp}vs d.
5. Compare with theoretical $C_{\mathrm{th}} = \varepsilon_0 A/d$. Plot C_{exp}vs $1/d$— should be linear; slope ≈ $\varepsilon_0 A$.

Method B — RC time constant (if no capacitance meter)

1. Charge the capacitor through a known resistor $R$from a DC supply V_0.
2. Measure charging (or discharging) voltage as a function of time using voltmeter/oscilloscope. For charging:

$$V(t) = V_0(1 - e^{-t/RC}).$$

Determine time constant $\tau = RC$by fitting data or measuring time to reach $0.632V_0$. Then compute $C = \tau/R$.

3. Repeat for different separations d.

Sample data layout (example)

Trial	**Plate area A(m²)**	**Separation d(m)**	**Measured C_{exp}(F)**	**Theoretical $C_{\mathrm{th}} = \varepsilon_0 A/d$(F)**
1	0.0200	1.00×10^{-3}	...	$8.85 \times 10^{-12} \cdot 0.02/10^{-3}$

(Compute numerical values using measured numbers.)

Calculations and verification

- Compute percentage error:

$$\%\ \text{error} \ = \ \left| \frac{C_{\text{exp}} - C_{\text{th}}}{C_{\text{th}}} \right| \times 100\%.$$

- Plot C_{exp}vs $1/d$. Fit a straight line; slope = $\varepsilon_{\text{eff}}\, A$. From slope, estimate experimental permittivity ε_{eff}.

Precautions

- Keep plates perfectly parallel and surfaces clean.
- Use small voltages to avoid air ionization.
- Use a guard/shield or guard ring to reduce leakage and stray capacitance for precise measurements.
- Minimize connecting lead lengths and keep measurement instruments grounded correctly.
- Measure separation $d$accurately; errors in $d$produce large errors in C(since $C \propto 1/d$).
- If using RC method, ensure resistor is stable and time resolution is adequate.

Sources of experimental error

- Fringing fields (edge effects) not accounted for in ideal formula.
- Uncertainty in plate area A, plate separation d, and instrument precision.
- Parasitic (stray) capacitances from leads and instrument.
- Non-parallel alignment and surface irregularities.
- Leakage currents and dielectric absorption in air/humidity.

Plate area $A = 0.02\ \text{m}^2$(for example), separation $= 1.00 \times 10^{-3}$ m. Using $\varepsilon_0 = 8.854 \times 10^{-12}\ \text{F m}^{-1}$:

Compute theoretical capacitance:

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 0.02}{1.00 \times 10^{-3}} = 8.854 \times 10^{-10} \text{ F} = 885.4 \text{ pF}.$$

- For an ideal parallel-plate capacitor without dielectric, $C = \frac{\varepsilon_0 A}{d}$.
- The relation is derived from the uniform electric field between plates and the definition $C = Q/V$.
- Experimental verification requires careful control of plate geometry and measurement of small capacitances; fringing and stray capacitances are important practical considerations.

1.3 Effect of Dielectric

A **dielectric** is an insulating material placed between the plates of a capacitor. Its primary role is to change the electric field distribution and allow the capacitor to store more charge for the same applied potential difference. When a dielectric slab is inserted between the plates of a parallel-plate capacitor, the **capacitance increases**. This happens due to **polarization** of the dielectric medium.

Polarization of Dielectric

When an external electric field is applied across the plates:

1. Molecules of the dielectric get slightly displaced.
2. This creates **induced dipoles** inside the material.
3. These induced dipoles produce their own electric field **opposite** to the external field.

The net electric field inside the dielectric becomes:

$$E = \frac{E_0}{K}$$

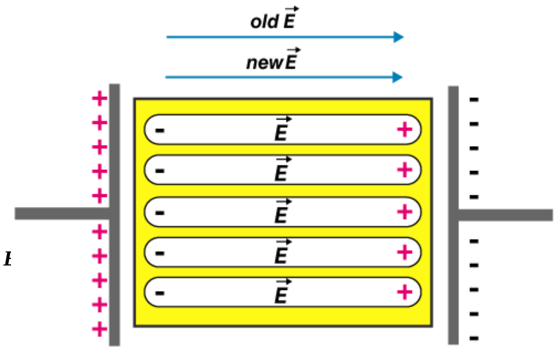


Figure 1. 2 (Dielectric Capacitor)

where

E_0= electric field without dielectric

K= dielectric constant (relative permittivity)

Because the electric field reduces, the potential difference for the same charge Q decreases.

Capacitance with Dielectric (Full Insertion)

If a dielectric of constant Kcompletely fills the region between plates:

$$C = K\, C_0$$

where

$$C_0 = \frac{\varepsilon_0 A}{d}$$

is the capacitance without dielectric.

Thus,

$$C = \frac{K\varepsilon_0 A}{d}$$

This shows capacitance increases linearly with the dielectric constant.

Typical dielectric constants:

- Air ≈ 1
- Glass ≈ 5–10
- Mica ≈ 7
- Porcelain ≈ 6
- Plastic ≈ 2–4
- Water ≈ 80

Higher $K \Rightarrow$ higher capacitance.

Effect on Electric Field and Potential

Electric Field With Dielectric

$$E = \frac{E_0}{K}$$

The field inside is reduced because of polarization.

Potential Difference

Potential between plates:

$$V = E\,d = \frac{E_0 d}{K}$$

Thus,

$$V = \frac{V_0}{K}$$

where $V_0$is the potential without a dielectric.

Charge Storage

For a fixed applied voltage V:

$$Q = CV = KC_0V$$

→ capacitor stores **K times more charge**.

Energy Stored in a Capacitor with Dielectric

Energy without dielectric:

$$U_0 = \frac{1}{2}C_0V^2$$

Energy with dielectric:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}KC_0V^2$$

→ Energy stored increases when voltage is constant.

But if charge is constant:

$$U = \frac{Q^2}{2C}$$

Since Cincreases by K,

energy **decreases** by $1/K$.

So:

- **Constant V:** energy increases
- **Constant Q:** energy decreases

Dielectric Partially Filling the Space

If the dielectric slab does not completely fill the space, effective capacitance is:

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

where

t= thickness of dielectric slab

d= plate separation

This formula is important in practical capacitors.

Breakdown Strength of Dielectric

Every dielectric has a **breakdown field strength**, above which it becomes conductive.

High dielectric strength materials allow capacitors to operate at high voltages safely.

Examples:

- Air: 3×10^6 V/m
- Mica: 120×10^6 V/m
- Glass: 40×10^6 V/m

A dielectric with high breakdown strength → capacitor can hold higher voltages → greater energy density.

Property	With Dielectric
Capacitance	Increases by factor (K)
Electric field	Reduced by (1/K)
Potential difference	Reduced for same charge
Charge storage	Increases for same voltage
Energy stored	Depends on constant Q or constant V
Breakdown voltage	Increases
Polarization	Induced dipoles reduce effective field

1.4 Carey Foster Bridge

The **Carey Foster Bridge** is a modified form of the Wheatstone bridge designed specifically for **measuring very small differences between two nearly equal resistances**. Instead of giving the absolute value of a resistance, it lets you find how much one resistor differs from another with extremely high accuracy. This is why it's used in precision labs to determine **low-resistance coils, shunt resistances, and small resistance variations**.

Construction

The Carey Foster Bridge consists of: (As in the figure 1.3)

1. A **uniform resistance wire** (usually 1 m long) with known resistance per unit length.
2. A **Wheatstone-bridge arrangement** with four arms.
3. A **galvanometer** connected between the middle points of the two opposite arms.
4. A **standard known resistor (R_1)** and a **variable balancing resistor (R_2)**.
5. Two resistances under comparison:
   - **X** — the unknown resistance
   - **Y** — the known or reference resistance
6. A sliding contact (jockey) to move along the wire.

The wire is mounted on a scale so that exact balancing length can be measured.

Working Principle

The bridge works on the **null-balance principle** of the Wheatstone bridge:

$$\frac{R_1}{R_2} = \frac{X + r_l}{Y + r_r}$$

where

- r_l, r_r are the resistances of the left and right segments of the bridge wire.

Instead of solving complicated ratios, Carey Foster's method **shifts the position of X and Y** and compares the balancing lengths.

Procedure (Conceptual)

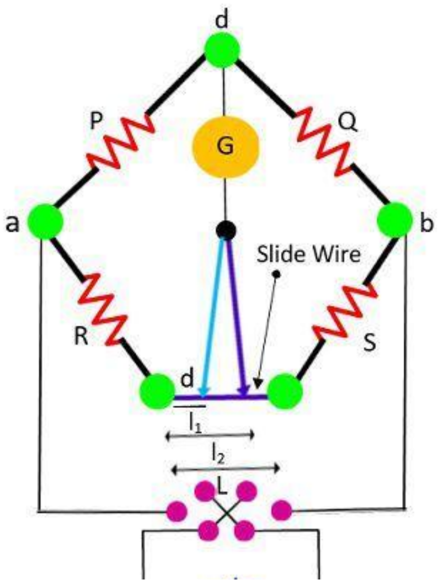


Figure 1. 3 (Caray Foster Bridge)

1. Insert **X** in the left gap and **Y** in the right gap.
2. Adjust balance point on the wire — say the length from left end is l_1.
3. Now *interchange* X and Y.
4. Again adjust the balance point — say the new length is l_2.

The key observation is:

When you swap the resistors, the difference in balancing lengths depends ONLY on the **difference between X and Y**, not on the rest of the bridge.

Derivation for Resistance Difference

Let

- ρ= resistance per unit length of the bridge wire.
- l_1= balancing length when X is on left.
- l_2= balancing length after interchanging X and Y.

Then the **difference between the two resistances** is given by:

$$X - Y = \rho(l_1 - l_2)$$

This is the core formula of the Carey Foster Bridge.

No matter how messy the remaining parts of the bridge are, the difference only depends on the wire segment.

- To measure **very small resistance differences** with high accuracy.
- To calibrate low-value resistors (in milliohm range).
- Useful in precision labs and metrology centers.

Because $l_1 - l_2$can be measured very accurately, even tiny resistance differences (0.001 Ω or less) can be detected.

Advantages

- High sensitivity
- Null method → no error due to galvanometer resistance
- Accurate even for very low resistances
- Temperature effects are minimized because wire resistance is uniformly distributed

Limitations

- Requires a uniform resistance wire
- Not suitable for large resistance differences
- Needs precise balancing and steady observation
- Accuracy depends on how perfectly the wire is calibrated

Result

$$X - Y = \rho(l_1 - l_2)$$

Where:

- X= unknown resistance
- Y= known resistance
- ρ= resistance per unit length of the bridge wire
- l_1, l_2= balance lengths before and after interchanging X and Y

Temperature of Coefficient of Resistance

The **temperature coefficient of resistance** is a physical constant that tells you **how much the resistance of a material changes when its temperature changes**. Most electrical conductors (like metals) have resistance that **increases** with temperature. Most semiconductors and insulators show the **opposite** behaviour. The **temperature coefficient of**

resistance, denoted by **α**, is defined as the **fractional change in resistance per degree rise in temperature**.

Formally,

$$\alpha = \frac{R_t - R_0}{R_0(t - 0)}$$

where

- R_0= resistance at $0°C$
- R_t= resistance at temperature $t°C$

This basically says:

"How much does resistance change for every 1°C rise?"

For moderate temperature ranges, the resistance of a conductor varies **linearly** with temperature:

$$R_t = R_0(1 + \alpha t)$$

This is the main working formula.

For a change in temperature:

- $R_2 = R_1[1 + \alpha(T_2 - T_1)]$

  If **α > 0** → resistance increases with temperature (metals).
- If **α < 0** → resistance decreases with temperature (semiconductors).
- If **α ≈ 0** → resistance is nearly independent of temperature (manganin, constantan).

So α basically tells the "temperature behaviour" of the material.

Metals (positive α):

When temperature rises, atoms vibrate more → electrons collide more → resistance increases.

Hence **positive TCR**.

Semiconductors (negative α):

Temperature increases → more electrons jump to conduction band → conductivity increases → resistance decreases. Hence **negative TCR**.

Material	α (per °C)
Copper	0.0043
Aluminium	0.004
Iron	0.006
Tungsten	0.0045
Manganin	~0
Constantan	~0
Silicon	Negative
Germanium	Negative

Materials with almost zero α are used for **standard resistors** because their resistance stays stable.

Importance of TCR

- Used to design **temperature sensors (RTDs)**
- Helps in **precision circuits** where stable resistance is required
- Essential in **calibrating resistors**
- Needed in **power electronics** for thermal stability
- Helps predict **heat losses** in metallic conductors

Summary

- Temperature coefficient α tells how resistance changes with temperature.
- Linear relation: $R_t = R_0(1 + \alpha t)$.
- Metals → α positive.
- Semiconductors → α negative.

- Alloys like manganin → α nearly zero → stable resistors.

Seebeck Effect

Introduction

When two different metals or semiconductors are joined together to form a closed circuit and the two junctions are maintained at *different temperatures*, an **electric current** begins to flow in the circuit even without any external battery. This remarkable conversion of **heat directly into electricity** was first discovered by **Thomas Johann Seebeck (1821)**. This phenomenon forms the foundation of modern **thermocouples**, **thermoelectric generators**, and temperature-sensing devices.

The **Seebeck effect** is defined as:

The phenomenon in which an electromotive force (emf) is produced in a closed circuit made of two dissimilar conductors when their junctions are kept at different temperatures.

The generated emf is called **thermoelectric emf**, and the circuit formed is called a **thermocouple**.

Metals have free electrons.

When one junction is heated:

- Electrons at the hot junction gain more kinetic energy.
- These energetic electrons diffuse towards the cold junction.

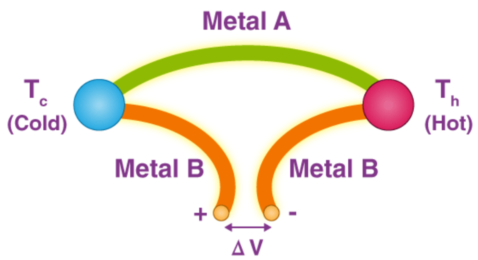


Figure 1. 4 (Seebeck Effect)

- This movement of charge creates a **potential difference** across the circuit.

The direction of current depends on the combination of metals used.

Thus, the Seebeck effect fundamentally arises from **temperature-dependent electron diffusion**.

Seebeck Coefficient

The emf produced is approximately proportional to the temperature difference:

$$E = S\Delta T$$

where,

- E= thermoelectric emf
- S= **Seebeck coefficient** (also called thermo emf coefficient)
- $\Delta T = T_{\text{hot}} - T_{\text{cold}}$

The Seebeck coefficient depends on:

- Nature of the materials
- Temperature range
- Purity and physical condition of metals

Typical values:

- Copper–Constantan: ~40 μV/°C
- Iron–Constantan: ~52 μV/°C
- Chromel–Alumel (K-type): ~41 μV/°C

Thermocouple

A thermocouple is a practical device based on the Seebeck effect.

It consists of:

- Two different conductors (e.g., Copper and Iron)
- Two junctions:
  - **Hot junction** (measurement point)
  - **Cold junction** (reference point)

The output emf is measured and calibrated to determine temperature.

Characteristics of Seebeck Effect

1. The emf depends on the **pair** of metals used.

2. The emf depends on the **temperature difference**, not on absolute temperatures.
3. The relation between emf and temperature is **non-linear** at large temperature differences.
4. The direction of emf reverses when hot and cold junctions are swapped.

Applications

- **Thermocouples** for temperature measurement (industry, laboratories).
- **Thermoelectric generators (TEGs)** for converting heat → electricity.
- Used in power plants, space probes (RTGs), automotive engines.
- Temperature sensing in ovens, furnaces, and aerospace instruments.

Summary

- Seebeck effect = generation of emf due to temperature difference.
- Based on electron diffusion from hot to cold junction.
- Used in thermocouples and thermoelectric power generation.
- Described by $E = S\Delta T$.
- Fundamental thermoelectric phenomenon.

Laws of Thermo emf

When two dissimilar conductors are joined to form a thermocouple, and the junctions are maintained at different temperatures, an electromotive force (emf) is produced in the circuit. This emf is known as **thermo-emf**, and it arises mainly due to the **Seebeck effect**. The magnitude and direction of this thermo-emf depend on:

- the **nature of the two metals**, and
- the **temperatures of the hot and cold junctions**.

To understand and predict the behaviour of thermo-emf in different combinations of metals, certain fundamental principles known as the **Laws of Thermoelectricity** or **Laws of**

Thermo-emf were established. These laws are crucial in developing thermocouples, thermoelectric circuits, and industrial temperature-measurement systems.

There are **three** main laws:

1. Law of Homogeneous Circuits
2. Law of Intermediate Metals
3. Law of Intermediate Temperatures

Law of Homogeneous Circuits

Statement

No thermoelectric emf is produced in a circuit made of a **single homogeneous metal**, even if the circuit has temperature differences at various points.

Explanation

A thermocouple requires **two different metals**.

If the entire closed circuit is made of the same metal (e.g., only copper):

- Temperature differences along the metal do not produce any net Seebeck emf.
- Electrons move within the same lattice structure, so no junction potential is formed.

Thus **no thermo-emf** and **no thermoelectric current** can exist.

Significance

- A thermocouple must be made of **two dissimilar conductors**.
- Any measurement using a single metal conductor cannot produce Seebeck emf.

Law of Intermediate Metals

Statement

If a third metal is inserted between two metals of a thermocouple, the total thermo-emf of the circuit **remains unchanged**, provided the **junctions with the third metal are at the same temperature**.

$$E_{AB} = E_{AC} + E_{CB}$$

Explanation

Suppose you have a thermocouple made of metals **A** and **B**.

If you insert a third metal **C** between them (A → C → B), then:

- As long as the new junctions A–C and C–B are at **identical temperatures**,
- no additional thermo-emf is created.

Thus, the third metal **does not affect** the net thermoelectric emf of the original pair A–B.

Practical Importance

- This law allows us to connect **measuring instruments** (galvanometer, voltmeter) made of different metals to a thermocouple.
- Copper–constantan thermocouples can be connected using copper wires without creating extra emf.

Law of Intermediate Temperatures

Statement

The thermo-emf of a thermocouple with its junctions at temperatures $T_1$and $T_3$is equal to the algebraic sum of the thermo-emfs when the junctions are at T_1, $T_2$and at T_2, T_3.

$$E(T_1, T_3) = E(T_1, T_2) + E(T_2, T_3)$$

Explanation

Consider a thermocouple of metals A and B.

If the hot and cold junction temperatures are $T_1$and T_3, we can imagine splitting the temperature difference through an intermediate temperature T_2.

The total thermo-emf is **additive** across temperature intervals.

This law indicates that:

- Thermo-emf is a **function of temperature difference**, not absolute temperature.
- The emf–temperature graph is continuous and predictable.

Uses

- Enables calibration of thermocouples at different temperature ranges.
- Helps in constructing **thermoelectric charts** and tables.
- Allows stepwise computation of emf for large temperature gaps.

Law	Statement	Key Idea	Practical Use
Law of Homogeneous Circuits	A single metal circuit cannot produce thermo-emf	Two metals needed	Explains why thermocouples must use dissimilar metals
Law of Intermediate Metals	Introducing a third metal does not change emf if junctions are at same temperature	Junctions at same temperature → no extra emf	Allows connection to external instruments
Law of Intermediate Temperatures	$emf(T_1, T_3) = emf(T_1, T_2) + emf(T_2, T_3)$	emf is temperature-dependent and additive	Temperature calibration, thermocouple tables

Peltier effect

The **Peltier effect** is a thermoelectric phenomenon in which heat is either **absorbed** or **evolved** at the junction of two dissimilar conductors when an electric current passes through the junction. Discovered by Jean Charles Athanase Peltier in 1834, the effect is the electrical converse of the Seebeck effect: Seebeck converts a temperature difference to an emf, while Peltier converts an electrical current to heat transfer at junctions. The Peltier effect provides the physical basis for **solid-state cooling and heating devices** (thermoelectric coolers or Peltier modules).

Qualitative physical picture

Free charge carriers (electrons or holes) in a conductor carry not only charge but also energy (enthalpy). Different materials have different carrier energy distributions (chemical potential and enthalpy per carrier). When carriers pass from material A into material B across their junction, they undergo a change in their energy content. That change appears as heat:

- If carriers moving across the junction **lose** energy, heat is **evolved** at the junction (junction warms).
- If carriers **gain** energy, heat is **absorbed** at the junction (junction cools).

Thus, by controlling the direction and magnitude of current, one can make a junction absorb heat (cool) or evolve heat (heat).

Quantitative statement (basic law)

For a junction between two materials A and B carrying an electric current I, the rate of heat absorbed (or evolved) at the junction is

$$\dot{Q}_{AB} = \Pi_{AB}\, I$$

where

- $\dot{Q}_{AB}$ is the heat per unit time at the A–B junction (watts).
- I is the electric current (amperes), taken positive in a chosen direction.

- Π_{AB} is the **Peltier coefficient** of the junction A–B (units: volt, V, or equivalently J C $^{-1}$).

If $\Pi_{AB} > 0$ and current I flows from A to B, heat $\Pi_{AB} I$ is absorbed at the junction (or released — sign convention must be observed). More generally,

$$\Pi_{AB} = \Pi_B - \Pi_A,$$

with Π_A, Π_B the absolute Peltier coefficients of materials A and B. Therefore the heat at an A–B junction can be written as the difference of material Peltier coefficients times current.

Units: Π has units J/C = V. So ΠI has units (J/C)·A = J/s = W.

Sign convention and interpretation

- If $\dot{Q}_{AB} > 0$: heat **absorbed** at the junction (cooling).
- If $\dot{Q}_{AB} < 0$: heat **released** at the junction (heating).

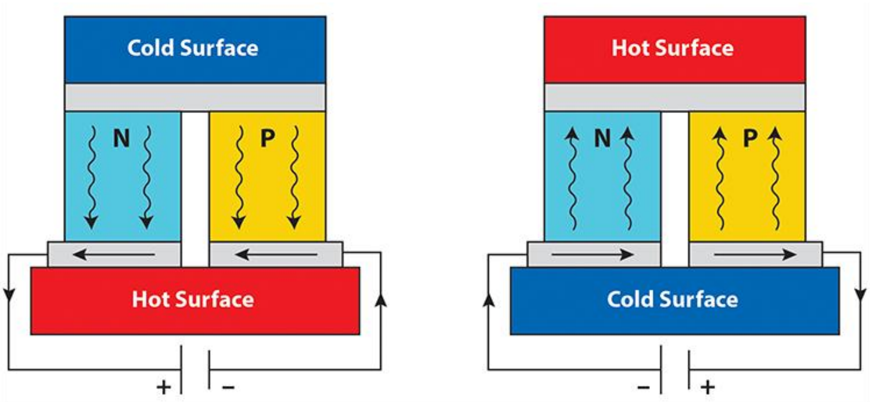


Figure 1. 5 (Peltier Effect)

- Reversing current direction reverses the sign of $\dot{Q}_{AB}$.

Kelvin (Thomson) relation between Seebeck and Peltier coefficients

Thermodynamic (reversible) considerations give a fundamental relation between the Seebeck coefficient S(units V K $^{-1}$) and the Peltier coefficient Π:

$$\boxed{\Pi = S\,T}$$

where $T$is the absolute temperature (in kelvin) at which the relation holds. For a junction A–B:

$$\Pi_{AB}(T) \;=\; T\,S_{AB}(T)$$

where $S_{AB} = S_B - S_A$is the Seebeck coefficient difference for the pair. This **Kelvin relation** is a direct consequence of Onsager reciprocity and reversible thermodynamics; it links the two thermoelectric effects (Seebeck and Peltier) and ensures consistency with the second law.

Consider a segment of conductor carrying current I. The local energy balance per unit time includes:

1. **Peltier heat** at junctions (localized).
2. **Joule heating** (resistive losses) distributed along the material: $\dot{Q}_{Joule} = I^2 R$.
3. **Heat conduction** along the material (Fourier conduction).

In a simple two-leg thermoelectric pair (A and B), total heat at the two junctions equals the sum of Peltier contributions plus Joule heating and conducted heat. In practical Peltier coolers, one designs legs so that at the cold junction Peltier absorption outweighs Joule heating and heat conduction into the cold side, producing net cooling.

Microscopically, the Peltier coefficient Πequals the enthalpy (energy per charge) transported by carriers relative to a chosen reference. When a carrier crosses from A to B, the enthalpy change per unit charge is Π_{AB}, giving heat $\Pi_{AB} I$. This viewpoint explains why material properties (carrier concentration, band structure) strongly control Π.

A practical Peltier cooler is an assembly of many p-type and n-type semiconductor legs electrically in series and thermally in parallel, sandwiched between ceramic plates. Current passing through the series legs causes one face to become cold (heat absorbed at cold junctions) and the opposite face to become hot (heat released), enabling active refrigeration without moving parts.

Key performance parameters:

- **Cooling power** (watts) — how much heat can be pumped at a given current.
- **ΔTmax** — maximum achievable temperature difference between hot and cold faces (occurs at zero heat load).
- **Coefficient of performance (COP)** — ratio of heat removed to electrical power input (lower than vapor-compression cycles for many cases).

Figure of merit and efficiency

Thermoelectric materials are characterized by a dimensionless **figure of merit** ZT:

$$ZT = \frac{S^2 \sigma T}{\kappa}$$

where

- S= Seebeck coefficient (V K^{-1}),
- σ= electrical conductivity (S m^{-1}),
- κ= thermal conductivity (W m^{-1} K^{-1}),
- T= absolute temperature (K).

Higher $ZT \rightarrow$ better thermoelectric performance (higher efficiency). Practical materials for Peltier devices are optimized to have large S, high σ, and low κ.

Applications

- **Solid-state cooling** (portable refrigerators, laser diode coolers, CCD/IR detector cooling, electronic component thermal management).
- **Temperature stabilization** (precision instruments).
- **Heat pumping and spot cooling** in aerospace and medical devices.
- **Energy harvesting** in reverse mode (thermoelectric generators use Seebeck but the same materials are relevant).

Advantages and limitations

Advantages

- No moving parts → high reliability, low maintenance.
- Compact, vibration-free, precise temperature control.
- Works at small scales and for low temperature differentials.

Limitations

- Low thermodynamic efficiency relative to conventional refrigeration (COP is modest).
- Performance strongly depends on material ZT; many materials have $ZT \ll 1$ at room temperature.
- Joule heating and thermal conduction reduce net cooling; careful device design required.

Worked numerical example

If a junction between materials A and B has $\Pi_{AB} = 0.03$ Vand a current $I = 2.0$ Apasses from A to B, the heat absorbed per second at the junction is

$$\dot{Q}_{AB} = \Pi_{AB}\, I = 0.03 \times 2.0 = 0.06 \text{ W}.$$

If current is reversed, the junction will release 0.06 W of heat instead.

(This example illustrates units and scaling; real Peltier modules use many junctions to reach practical cooling powers of several watts to hundreds of watts.)

Experimental observation and measurement

To observe Peltier heating/cooling:

- Build a simple junction of two dissimilar conductors (or use a commercial Peltier device).
- Apply a steady DC current and measure temperature change at the junction using a thermocouple or RTD.
- Separate Peltier effect from Joule heating by careful experimental design (use low resistance leads, measure as function of current and look for linear vs quadratic dependence: Peltier $\propto I$, Joule $\propto I^2$).

Relation to Seebeck and Thomson effects (thermoelectric family)

- **Seebeck effect:** temperature difference → emf (S).
- **Peltier effect:** current → heat at junction (Π).
- **Thomson effect:** current in a single conductor with a temperature gradient produces or absorbs heat continuously along its length (coefficient τ).

These three are thermodynamically linked; Kelvin relations connect them (e.g., $\Pi = ST$, and the Thomson coefficient $\tau = T\,\frac{dS}{dT}$).

Summary (key points)

- Peltier effect: heat absorbed or evolved at a junction when current flows; quantified by $\dot{Q} = \Pi I$.
- Peltier coefficient Πhas units of volts (J per coulomb).
- Kelvin relation ties Πto the Seebeck coefficient: $\Pi = ST$.
- Basis of thermoelectric coolers; practical performance depends on material figure of merit ZT.
- Distinguish linear Peltier term ($\propto I$) from Joule heating ($\propto I^2$) in experiments and device design.

Thomson Effect

The Seebeck and Peltier effects describe thermoelectric phenomena at junctions of two dissimilar conductors. In 1851, William Thomson (later Lord Kelvin) extended this understanding by discovering that **even a single homogeneous conductor** can exhibit heating or cooling when an electric current flows through it under a temperature gradient. This phenomenon is known as the **Thomson effect** and forms the third fundamental thermoelectric effect, completing the relations between Seebeck, Peltier, and Thomson processes.

The Thomson effect is defined as: The heating or cooling that occurs **continuously along the length of a single conductor** when an electric current flows through it and the conductor is maintained at different temperatures. Unlike the Peltier effect (which occurs only at junctions), the Thomson effect occurs **within** the body of a material.

- If a conductor has one end hotter than the other, the energy of electrons changes continuously along its length. When current flows:
- If the current carries carriers from a region of lower energy to higher energy, the conductor must absorb heat to support that energy increase → cooling.

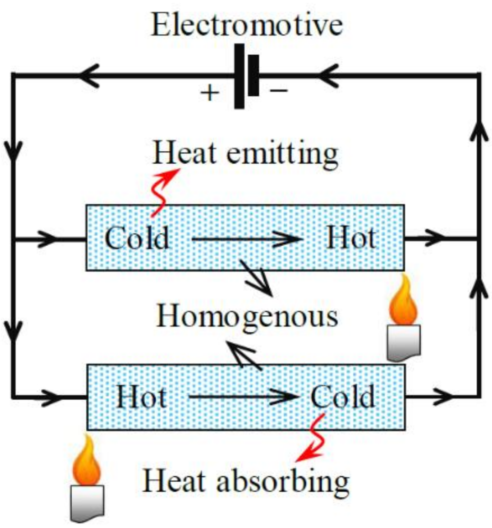


Figure 1. 6

- If carriers move from higher to lower energy regions, they release excess energy as heat → heating.

Thus, depending on the direction of temperature gradient and current, the conductor either absorbs or releases heat at every point.

Thomson Coefficient

The rate of heat absorbed or evolved per unit length in the conductor is written as:

$q = \tau I \, dT/dx$

where

q = Thomson heat per unit length per unit time

τ = Thomson coefficient of the material

I = current

dT/dx = temperature gradient along the conductor

τ is the key thermodynamic property and differs for different materials.

Sign and Meaning

- If τ is positive → heat is absorbed when current flows from cold to hot end.
- If τ is negative → heat is released when current flows from cold to hot end.

Examples:

- For iron, τ is positive.
- For copper, τ is negative.
- For lead, τ is nearly zero (no Thomson effect).

Difference from Joule Heating

Thomson heating is **reversible** and depends on the direction of current.

Joule heating is **irreversible** and always produces heat (I^2R).

Total heat in a conductor is therefore:

Total heat = Joule heat (I^2R) + Thomson heat ($\tau I \, dT/dx$)

The presence of both makes practical thermoelectric devices complex.

Kelvin (Thomson) Relations

Kelvin derived a connection between Seebeck, Peltier, and Thomson coefficients using thermodynamic laws.

Two important relations are:

1. $\Pi = S\,T$

   (Peltier coefficient = Seebeck coefficient × temperature)

2. $\tau = T\,dS/dT$

   (Thomson coefficient equals temperature times variation of Seebeck coefficient with temperature)

These relations show that the three effects are not independent but thermodynamically linked.

Experimental Observation

To observe the Thomson effect:

• Take a long conductor with a controlled temperature gradient.

• Pass a steady current through it.

• Measure temperature changes along the conductor.

• Reverse the current direction; the sign of Thomson heating reverses.

This reversible behavior confirms the existence of the effect.

Applications

The Thomson effect is not commonly used directly in devices but is crucial for:

• Designing accurate thermocouples

• Modeling thermoelectric cooling and power generation

• Understanding energy transport in semiconductor devices

• Calibration of high-precision temperature sensors

Summary

• Thomson effect occurs in a single conductor with a temperature gradient.

• It produces heating or cooling depending on current direction.

• Quantified by Thomson coefficient τ.

• It is reversible unlike Joule heating.

• Kelvin relations link it to Seebeck and Peltier effects.

Thermoelectric Diagrams and their uses

Thermoelectric phenomena arise from the interaction between heat and electricity in conductors and semiconductors. These effects — Seebeck, Peltier, and Thomson — form the foundation of modern thermoelectric devices used for temperature measurement, cooling, and power generation. To understand these effects clearly, physics textbooks employ a series of standard diagrams that visually represent the flow of current, movement of heat, temperature gradients, and junction behaviors.

Thermoelectric diagrams are therefore essential tools for illustrating how thermal and electrical energies convert into one another. They provide simplified yet accurate visual representations of practical systems such as thermocouples, Peltier coolers, and thermoelectric generators. Each diagram highlights a specific phenomenon: the development of thermo-emf in Seebeck circuits, heat absorption or evolution at junctions in the Peltier effect, and continuous heating or cooling along conductors in the Thomson effect.

In addition, diagrams showing semiconductor thermoelectric modules, thermoelectric series, and temperature-dependent coefficients help in understanding material behavior and device performance. These diagrams not only support theoretical learning but also guide practical applications in industries, laboratories, power systems, and electronic cooling technology.

1. Thermocouple Circuit Diagram

• Two dissimilar metals A and B joined at two junctions.

• One junction is kept hot, the other cold.

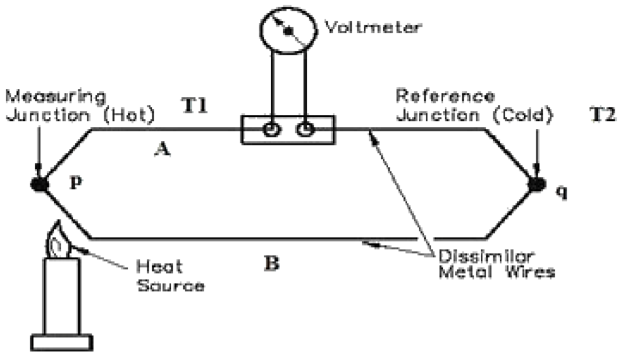


Figure 1. 7

• A galvanometer or voltmeter is connected in the loop.

Uses

• Measures temperature difference using Seebeck effect

• Forms the basis for industrial thermocouples

• Used in furnaces, exhaust systems, engines, labs, sensors

Why this diagram is important:

It visually shows how a temperature difference alone produces emf — no battery needed.

2. Peltier Effect Diagram

• Same two metals A and B joined at a junction.

• A battery is connected, driving current from A → B or B → A.

• The junction either absorbs or releases heat depending on current direction.

Uses

• Explains solid-state cooling

• Basis of Peltier coolers (TEC modules)

• Used in CPU coolers, camera sensors, small refrigerators, laser stabilization

Why this diagram is important:

It shows how current direction controls heating or cooling at the junction.

3. Thomson Effect Diagram

• A single conductor rod with one end hot and the other cold

• A current is passed through the rod

• Temperature markers show heating/cooling along the length

Uses

• Helps understand heat absorption/evolution inside a material, not just at junctions

• Important in precise temperature measurement

• Used in deriving Kelvin relations and thermodynamic consistency

Why this diagram is important:

Students usually think thermoelectric effects happen only at junctions. This diagram proves that a single conductor can also heat or cool internally due to a temperature gradient.

4. Thermoelectric Module (Peltier Cooler) Diagram

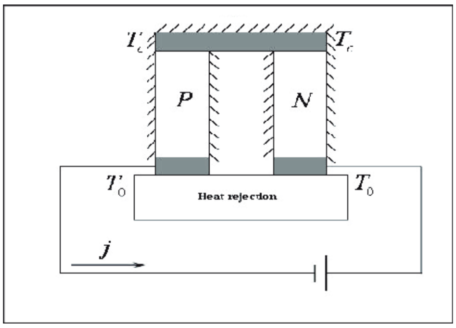


Figure 1. 8

• Alternating n-type and p-type semiconductor legs

• Electrically in series, thermally in parallel

• Cold plate on one side, hot plate on the other

• Direction of current decides which side gets cold

Uses

• Used in real cooling modules (TEC1-12706, etc.)

• Cooling of microchips, CCD sensors, portable mini-fridges

• Thermal stabilization of scientific instruments

Why this diagram is important:

This is the practical implementation of Peltier effect — how hundreds of junctions build a powerful cooler.

5. Thermoelectric Generator (TEG) Diagram

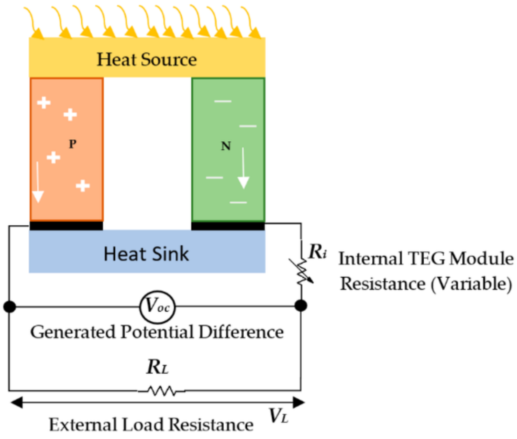


Figure 1. 9

• Hot surface at the top, cold surface at the bottom

• n-type and p-type legs connected

• Load (external circuit) connected to produce power

• Heat flows from hot → cold, generating current

Uses

• Converts waste heat into electricity

• Used in space probes (RTGs), automotive waste heat recovery, camping stoves

• Common in renewable energy and low-voltage power systems

Why this diagram is important:

Helps understand Seebeck effect in reverse — heat becomes electricity.

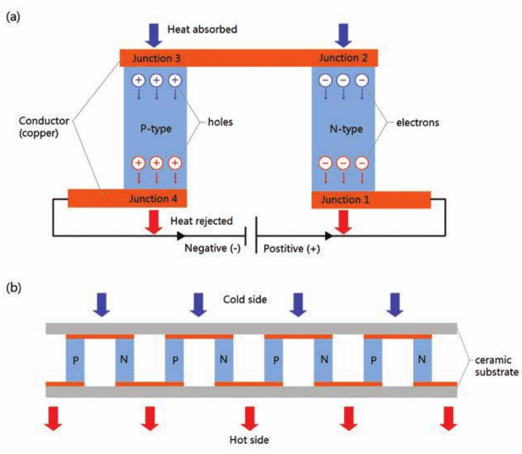


Figure 1. 10

Thermoelectric Series Diagram

• List of metals arranged in increasing order of thermoelectric power

• Arrows showing direction of thermoelectric emf when two metals are paired

Uses

• Helps choose thermocouple pairs (Copper–Constantan, Chromel–Alumel)

• Predicts direction and magnitude of emf

7. Seebeck Coefficient vs Temperature Graph

(Diagram Description)

• Graph of Seebeck coefficient (S) on y-axis vs temperature (K) on x-axis

• Each material has its own curve

Uses

• Shows that S varies with temperature

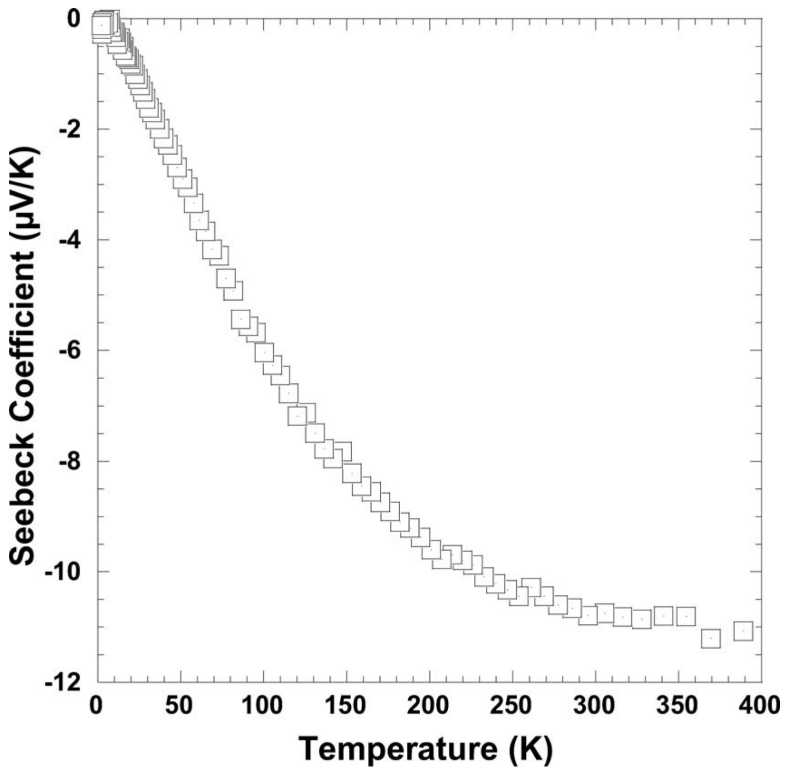


Figure 1. 11

• Helps calculate emf for thermocouples

• Used in material design for thermoelectric modules

8. Peltier Coefficient vs Temperature Graph

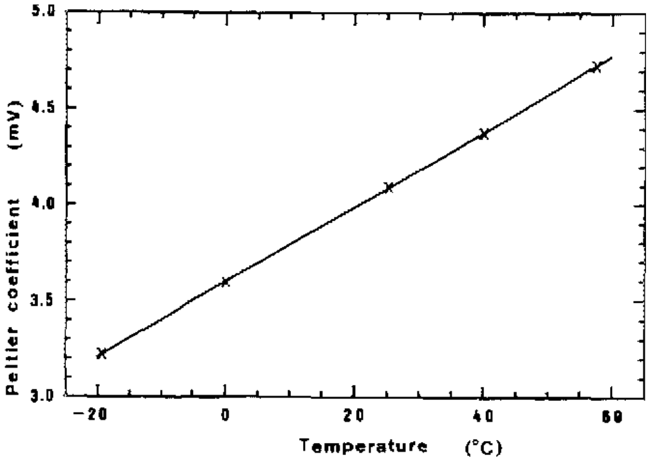


Figure 1. 12

• Π on y-axis vs T on x-axis

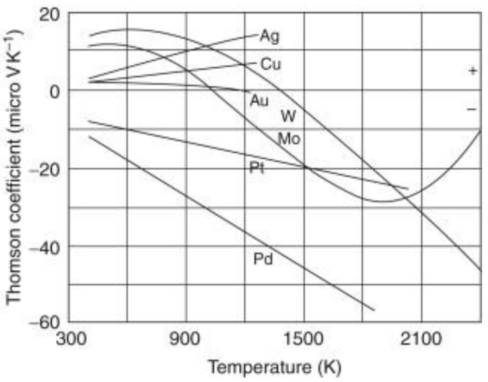
• Related directly by Π = S·T

Uses

• Helps understand cooling capability at different temperatures

• Important in designing Peltier devices

9. Thomson Coefficient vs Temperature Graph



• τ on y-axis vs T on x-axis

• Indicates materials with positive/negative Thomson behavior

Uses

• Used in calculating heat distribution along a conductor

• Needed for studying advanced thermodynamics of conduction

Thermodynamics of Thermocouple.

Introduction

A thermocouple is a temperature-measuring device based on **thermoelectric effects**, primarily discovered by **Thomas Johann Seebeck** in 1821. When two different metals are joined at two junctions and a temperature difference exists between them, an electromotive force (emf) is produced in the loop. This principle forms the basis for thermocouples.

To understand *why* and *how* this emf is generated, we need to study the **thermodynamics** behind the thermoelectric phenomena. The thermodynamics of a thermocouple deals with how heat energy is converted into electrical energy at the microscopic and macroscopic levels, and how different thermoelectric coefficients arise from these processes.

Fundamental Thermoelectric Effects Relevant to Thermodynamics

A thermocouple involves **three primary thermoelectric effects**:

(1) Seebeck Effect

When two dissimilar metals A and B form a closed loop with two junctions at different temperatures T_1 and T_2, a thermoelectric emf E is produced.

This emf arises due to:

- Diffusion of charge carriers (electrons),

- Difference in Fermi levels of the materials,
- Temperature-dependent carrier kinetics.

(2) Peltier Effect

When current flows through a junction of two metals, heat is either absorbed or evolved at that junction.

Peltier heat:

$$\Pi_{AB} = \Pi_A - \Pi_B$$

where $\Pi$is the **Peltier coefficient**.

(3) Thomson Effect

A single conductor carrying current and having a temperature gradient absorbs or releases heat continuously along its length.

Thomson heat:

$$dq = \sigma I dT$$

where $\sigma$is the **Thomson coefficient**.

These effects are thermodynamically interrelated and obey the **Kelvin Relations**.

Thermodynamic Basis of Thermoelectric Phenomena

The thermodynamics of a thermocouple connects:

- **Heat flow**,
- **Charge flow**,
- **Temperature difference**,
- **Generated emf**.

From thermodynamic principles, every thermoelectric effect must obey energy conservation and entropy rules.

Free Energy and Emf

The emf generated in a thermocouple is essentially the **change in Gibbs free energy** per unit charge.

$$E = -\frac{dG}{dq}$$

Since Gibbs free energy of electrons varies with temperature, the emf between two metals A and B is:

$$E = \int_{T_1}^{T_2} \quad (S_A - S_B) dT$$

Here $S_A, S_B$are **thermoelectric powers (Seebeck coefficients)**.

Thus, emf directly arises from **entropy differences of charge carriers** in the two metals.

Seebeck Coefficient and Temperature Dependence

Every metal has a characteristic **Seebeck coefficient** S.

For a thermocouple AB:

$$E = \int_{T_1}^{T_2} (S_A - S_B)\, dT$$

If Svaries linearly with temperature,

$$S = \alpha + \beta T$$

then the emf becomes:

$$E = a(T_2 - T_1) + \frac{1}{2} b(T_2^2 - T_1^2)$$

where

$$a = (\alpha_A - \alpha_B)$$

$b = (\beta_A - \beta_B)$.

This quadratic relation explains why thermocouple emf vs. temperature graphs are often *parabolic*.

Peltier Heat and Free Energy

At a junction of metals A and B, current moves electrons from one Fermi level to another.

This change in energy level leads to heat absorption or evolution.

Peltier coefficient:

$$\Pi_{AB} = T(S_A - S_B)$$

This is the **first Kelvin relation**, derived from the second law of thermodynamics.

Thomson Heat as Continuous Distribution

Thomson effect is not at junctions but along the conductor.

Thomson coefficient:

$$\sigma = T\frac{dS}{dT}$$

This gives the **second Kelvin relation**:

$$\frac{d\Pi}{dT} = S + T\frac{dS}{dT}$$

Both Kelvin relations ensure:

- Energy conservation,
- Reversibility of thermoelectric processes,

- Predictability of emf.

Net Emf in a Thermocouple – Thermodynamic Summation

For a complete thermocouple loop:

$$E = E_{\text{Seebeck}} + E_{\text{Peltier}} + E_{\text{Thomson}}$$

But thanks to Kelvin relations, all these contributions can be represented purely in terms of Seebeck coefficient:

$$E = \int_{T_1}^{T_2} (S_A - S_B)\, dT$$

Thus, **Seebeck coefficient is the master variable**.

Thermodynamic Cycle of a Thermocouple

A thermocouple loop with junctions at $T_1$and T_2acts like a **heat engine**:

Step 1: Heat absorbed at the hot junction (Peltier heat).

Step 2: Electrons flow through conductors (Thomson redistribution).

Step 3: Heat released at the cold junction.

Step 4: Net electrical work is done due to the emf.

This is analogous to a **reversible Carnot-like thermodynamic cycle**, where:

- Heat input → Hot junction
- Work output → Emf
- Waste heat → Cold junction

Thus a thermocouple converts **thermal energy to electrical energy**.

Laws Governing Thermocouple Thermodynamics

(1) Law of Intermediate Temperatures

$$E(T_1, T_3) = E(T_1, T_2) + E(T_2, T_3)$$

Ensures emf is path-independent.

(2) Law of Intermediate Metals

Introducing a third metal does not affect emf if junction temperatures are unchanged.

Ensures thermodynamic consistency.

Practical Implications

- Thermocouples can measure a broad range of temperatures (−200°C to 1800°C).
- The emf produced is small (few mV), requiring sensitive instruments.
- The emf–temperature relation relies on accurate Seebeck coefficients.
- Stability depends on metallurgical purity and junction integrity.

Unit 2: Magnetic Effect of Current

1. Biot and Savart Law
2. Magnetic Induction due to circular coil
3. Force on a current element by Magnetic field
4. Force between two infinitely long Conductors
5. Torque on a current loop in a field
6. Moving Coil Galvanometer
7. Damping Correction
8. Ampere's Circuital law
9. Differential form
10. Divergence of magnetic field
11. Magnetic induction due to toroid

Biot and Savart Law

Introduction

In electromagnetism, one of the fundamental tasks is to determine the **magnetic field produced by electric currents**. Experiments by **Jean-Baptiste Biot** and **Félix Savart** in 1820 showed that a steady electric current generates a magnetic field in the surrounding space, and the strength of this magnetic field depends on specific measurable factors. From their experimental observations, they formulated a mathematical relation that connects the magnetic field to the current element responsible for it. This relation is known as the **Biot–Savart Law**. It plays a role similar to Coulomb's law in electrostatics.

Biot–Savart Law is especially powerful for calculating magnetic fields due to:

- Straight wires
- Circular loops
- Solenoids
- Current distributions

It is considered a *microscopic law* because it describes the field due to a very small segment of current — the **current element**.

Biot–Savart Law

The law states that:

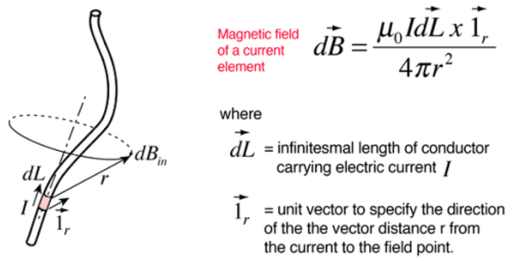
The magnetic field $d\vec{B}$ produced at a point due to a small current-carrying element is directly proportional to the current, the length of the element, and the sine of the angle between the element and the line joining the element to the point. It is inversely proportional to the square of the distance from the element to the point.

Mathematically

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}$$

where,

- I= current through the element
- $d\vec{l}$= vector length of the infinitesimal current element
- r= distance from the element to the point of observation
- $\hat{r}$= unit vector from element to observation point
- μ_0= permeability of free space



- $d\vec{l} \times \hat{r}$ensures direction using cross-product

Explanation of Each Term

1. Current Element $I d\vec{l}$

A tiny portion of a conductor carrying current is called a current element.

Its direction is along the direction of current flow.

2. Distance Term $\frac{1}{r^2}$

Just like Coulomb field decreases with distance squared, magnetic field from a small current element also decreases as r^{-2}.

3. Angular Dependence $\sin\theta$

$$dB \propto \sin\theta$$

The angle $\theta$is between the current element and the line joining it to the point.

- If $\theta = 0°$→ field = 0 (along the wire)
- If $\theta = 90°$→ maximum magnetic field

This explains why points directly in line with the current element don’t experience magnetic field.

4. Direction – Right-Hand Thumb Rule

The direction of the magnetic field is perpendicular to the plane formed by $d\vec{l}$and $\vec{r}$.

Given by:

Curl your right hand’s fingers in the direction of the magnetic field, with the thumb pointing in the direction of current.

Vector Form

The cross-product gives:

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l} \times \hat{r}}{r^2}$$

- Direction of $d\vec{B}$ is perpendicular to both $d\vec{l}$ and $\hat{r}$.
- Magnitude:

$$dB = \frac{\mu_0 I}{4\pi}\frac{Idl\sin\theta}{r^2}$$

Derivation of Expression

Biot and Savart formulated the law from experiments, not from first principles.

But Maxwell's equations later justified it theoretically.

Key assumptions:

1. Field is proportional to current I.
2. Field is proportional to element length dl.
3. Field diminishes as r^2.
4. Field depends on orientation (sinθ).
5. Direction follows right-hand rule.

When these observations are combined using vector algebra, the Biot–Savart equation emerges naturally.

Applications of the Law

1. **Magnetic field due to a long straight current-carrying conductor**

$$B = \frac{\mu_0 I}{2\pi r}$$

2. **Magnetic field at the center of a circular loop**

$$B = \frac{\mu_0 I}{2R}$$

3. **Magnetic field along the axis of a circular coil**

$$B = \frac{\mu_0 N I R^2}{2(R^2 + x^2)^{3/2}}$$

4. **Field distribution around shaped conductors**

This law is fundamental to electromagnetism and acts as the building block for Ampere's circuital law and Maxwell's equations.

Physical Interpretation

- The law shows that moving charges (current) produce a magnetic field.
- The magnetic field is stronger closer to the conductor.
- The distribution of field lines is circular around the wire.
- The pattern is always right-handed with respect to current direction.
- Biot–Savart law is similar to Coulomb's law but with vector cross-product because magnetic field is inherently rotational.

Magnetic Induction due to circular coil

A circular coil carrying electric current is one of the most important geometries in magnetism because it produces a well-defined magnetic field that is symmetric about its axis. Devices like galvanometers, motors, speakers, and MRI coils use this principle. When current flows through the circular loop, each tiny segment of the loop behaves like a current element and produces a magnetic field. Using the **Biot–Savart Law**, we can calculate the total magnetic induction at any point along the axis of the coil or at its center.

Consider a circular coil of:

- Radius = R
- Number of turns = N

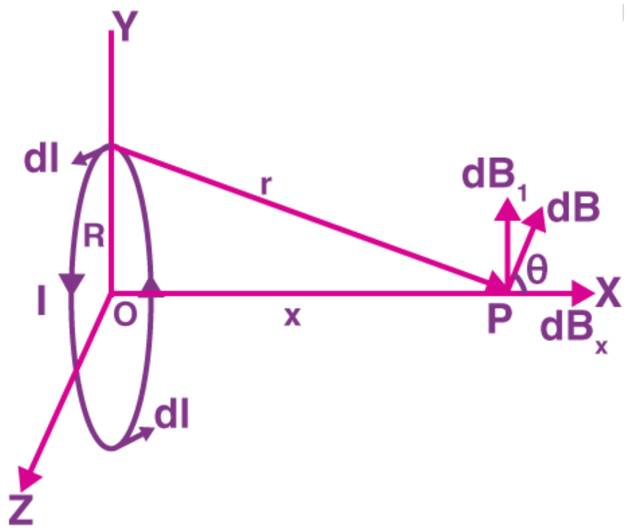


Figure 2. 1

- Current = I

By symmetry:

- Every current element produces magnetic field components.
- All **radial components cancel out**.
- Only **axial components add up**.

Using Biot–Savart Law:

- $dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin 90^\circ}{R^2}$

  Integrate around the full circle: $\int d\ell = 2\pi R$.

Thus

$$B = \frac{\mu_0 I}{4\pi R^2}(2\pi R) = \frac{\mu_0 I}{2R}.$$

For a coil with Nidentical turns (tightly wound),

$$\boxed{B_{\text{centre}} = \frac{\mu_0 N I}{2R}}$$

Direction: along the axis (use right-hand rule: thumb along current circulation → fingers show field direction).

Key points:

- Field is **directly proportional** to $N$and I.
- Field is **inversely proportional** to radius R.
- Direction is given by **Right-Hand Thumb Rule**.

where $\vec{r}$is the vector from the element to P, $r = |\vec{r}|$, and $\hat{r} = \vec{r}/r$.

Consider a circular loop of radius Rcarrying current I. Take the loop centre as origin and the loop lying in the xy-plane. The axis of the loop is the z-axis. Let Pbe on the axis at coordinate $z = x$

Magnetic field at a point on the axis (distance x from centre)

Take a generic current element $I\, d\vec{\ell}$at some position on the loop. Let the distance from this element to the point P(on the axis at $z = x$) be

$$r = \sqrt{R^2 + x^2}.$$

Important geometric facts:

- The tangent element $d\vec{\ell}$is perpendicular to the radius vector from the element to the loop centre, and the angle between $d\vec{\ell}$and $\vec{r}$is 90°. So $|d\vec{\ell} \times \hat{r}| = d\ell$.
- The magnetic field vector $d\vec{B}$produced by that element is perpendicular to the plane containing $d\vec{\ell}$and $\vec{r}$. That $d\vec{B}$has a component along the axis (call it dB_z) and a radial component that cancels by symmetry with the opposite element on the ring. So only axial components survive when integrating around the loop.

Compute the axial component:

1. Magnitude from Biot–Savart:

$$dB = \frac{\mu_0}{4\pi} \frac{I\, d\ell}{r^2}.$$

(since $\sin\theta = 1$for $\theta = 90°$).

2. Geometry gives the axial direction cosine:

$$\cos\phi = \frac{x}{r},$$

where ϕ is the angle between $\vec{r}$ and the axis.

3. So axial component:

$$dB_z = dB\cos\phi = \frac{\mu_0}{4\pi}\frac{I\,d\ell}{r^2}\cdot\frac{x}{r} = \frac{\mu_0 Ix\,d\ell}{4\pi r^3}.$$

4. Integrate $d\ell$ over the loop: $\int d\ell = 2\pi R$. Note that $r = \sqrt{R^2 + x^2}$ and is the same for every element on the loop (because all elements are same distance from point on axis).

Thus the total axial field at P is

$$B_z = \frac{\mu_0 Ix}{4\pi r^3}\,(2\pi R) = \frac{\mu_0 IxR}{2r^3}.$$

Replace $r = \sqrt{R^2 + x^2}$:

$$\boxed{B_{\text{axis}}(x) = \frac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}}}$$

(for a single turn). For N turns:

$$\boxed{B_{\text{axis}}(x) = \frac{\mu_0 NIR^2}{2(R^2 + x^2)^{3/2}}}$$

Checks / limits

- At $x = 0$ (centre): $B = \frac{\mu_0 NIR^2}{2(R^2)^{3/2}} = \frac{\mu_0 NI}{2R}$ — matches the centre result.
- As $x \to \infty$: $B \sim \frac{\mu_0 NIR^2}{2x^3}$ — field falls off as a dipole ($\propto 1/x^3$).

Direction: along the axis; sign determined by right-hand rule (if current is anticlockwise looking from +z, field at +z is +z direction).

Quick notes & intuition

- The factor $R^2/(R^2 + x^2)^{3/2}$controls how the field spreads: near the loop ($x \ll R$) field is strong; far away ($x \gg R$) it decays like a dipole.
- For a coil with many tightly-packed turns, multiply by N(superposition principle).
- Helmholtz coils use two identical coils separated by $R$to create a region of very uniform field around the midplane — useful for experiments needing uniform B.

Force on a current element by Magnetic field

When a conductor carries electric current, the moving charges (electrons) experience a force if they enter a region containing a magnetic field. This is a fundamental phenomenon in electromagnetism and forms the working principle of:

- Electric motors
- Moving-coil galvanometers
- Loudspeakers
- Meters and relays
- Rail-guns and many electromechanical systems

This interaction was discovered by **Ampère** and is fully explained mathematically using the **Lorentz force law**.

A current is nothing but **moving charges**.

If a charge $q$moves with velocity $\vec{v}$in a magnetic field $\vec{B}$, the force on it is:

$$\vec{F} = q(\vec{v} \times \vec{B})$$

Now extend this idea to **a wire carrying many charges moving together** — that's where the force on a current element comes from.

Take a tiny piece of conductor of length $d\vec{l}$carrying current I.

- Direction of $d\vec{l}$: along current

- Magnitude of $d\vec{l}$: length of that small element

We want the force on this little segment when placed in magnetic field $\vec{B}$.

Derivation – Force on Current Element

Step 1: Start with Lorentz force

For a small number of charges $n$per unit volume, each of charge q, moving with drift velocity $\vec{v}_d$:

$$\vec{F} = nqAd\ell\,(\vec{v}_d \times \vec{B})$$

where

- A= cross-sectional area
- $d\ell$= length of the element

Step 2: Replace $nqv_d$with current

Current is defined as:

$$I = nqAv_d$$

So:

$$\vec{F} = I\,d\ell\,(\hat{\ell} \times \vec{B})$$

Step 3: Write it in vector form

Since $\vec{\ell} = \hat{\ell}\,d\ell$,

$$\boxed{\vec{F} = I\,d\vec{\ell} \times \vec{B}}$$

This is the **force on a current element in a magnetic field**.

Magnitude of the Force

$$F = I\,d\ell\,B\sin\theta$$

where

- I= current
- $d\ell$= length of the current element
- B= magnetic field
- θ= angle between current direction and magnetic field

1. Current parallel to magnetic field

$$\theta = 0°, \sin\theta = 0$$

$$F = 0$$

No force — because moving charges don't get deflected if moving parallel to field lines.

2. Current perpendicular to magnetic field

$$\theta = 90°, \sin\theta = 1$$

$$F_{\max} = I\, d\ell\, B$$

Maximum deflection — used in galvanometers and motor coils.

Force Direction Rule – Right-Hand Rule

- Thumb → direction of current I
- Fingers → magnetic field $\vec{B}$
- Palm → direction of force

(For negative charges, force direction flips.)

Force on an Entire Conductor (not just element)

If the conductor has finite length L:

$$\boxed{\vec{F} = I\,\vec{L} \times \vec{B}}$$

Physical Interpretation

- Current-carrying wires feel a sideways push in magnetic fields.

- That’s why motor coils rotate — magnetic force acts on opposite sides in opposite directions.
- Loudspeaker voice coils vibrate by this force.

Applications

- DC motors
- AC motors
- Galvanometers
- Ampere’s force between parallel wires
- Railgun acceleration
- Magnetic levitation setups

Force between two infinitely long Conductors

Introduction

When electric current flows through a conductor, it produces a magnetic field around it. This fundamental principle is described by **Ampère’s Law** and is a key idea in electromagnetism. If **two long straight parallel conductors** carry currents, the magnetic field produced by one conductor interacts with the current in the other conductor. Because a current-carrying conductor experiences a force when placed in a magnetic field, **each conductor experiences a magnetic force due to the other**.

This force can be:

- **Attractive**, if the currents flow in the **same direction**
- **Repulsive**, if the currents flow in **opposite directions**

This physical phenomenon is so fundamental that the **SI unit of current (ampere)** is defined based on the force between two infinitely long parallel conductors.

Derivation of Force Between Two Long Parallel Conductors

Consider:

$$B_1 = \frac{\mu_0 I_1}{2\pi a} \qquad ...(1)$$

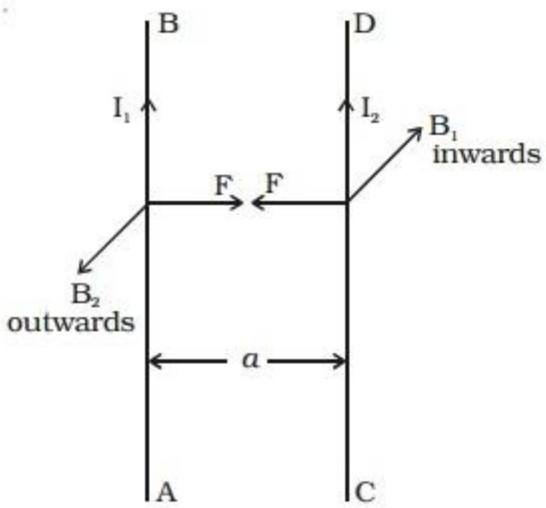


Figure 2. 2

- Two long straight conductors separated by a distance d
- Conductor 1 carries current I_1
- Conductor 2 carries current I_2
- Length of each conductor = L

Magnetic Field Created by Conductor 1 at the Position of Conductor 2

Using:

$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

This magnetic field is circular around conductor 1.

Force on Conductor 2 Due to This Magnetic Field

A conductor carrying current I placed in a magnetic field B experiences a force:

$$F = ILB$$

For conductor 2:

$$F_{21} = I_2 L B_1$$

Substitute B_1:

$$F_{21} = I_2 L (\frac{\mu_0 I_1}{2\pi d})$$

So the force is:

$$\boxed{F_{21} = \frac{\mu_0 I_1 I_2 L}{2\pi d}}$$

Force per Unit Length

Since the wires are very long (ideally infinite), we often find force **per unit length**:

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

This is the standard formula used in electromagnetism.

Direction of the Force

Using the right-hand rule:

- If currents I_1 and I_2 flow **in the same direction**,

  force is attractive.

- If currents flow in **opposite directions**,

  force is repulsive.

Definition of 1 Ampere

One ampere is defined as the current which, when maintained in two infinitely long, straight, parallel conductors placed 1 metre apart in vacuum, produces a force of $\mathbf{2 \times 10^{-7}}$ **N/m** on each conductor.

Torque on a current loop in a field

A current-carrying conductor placed in a magnetic field experiences a force. This force is described by the **Lorentz force law**. When the conductor is bent into a **closed loop**, such as a rectangular coil, the magnetic field exerts forces on different sides of the loop in different directions. These forces form a **couple**, producing **torque** that tends to rotate the loop.

This principle forms the basis of:

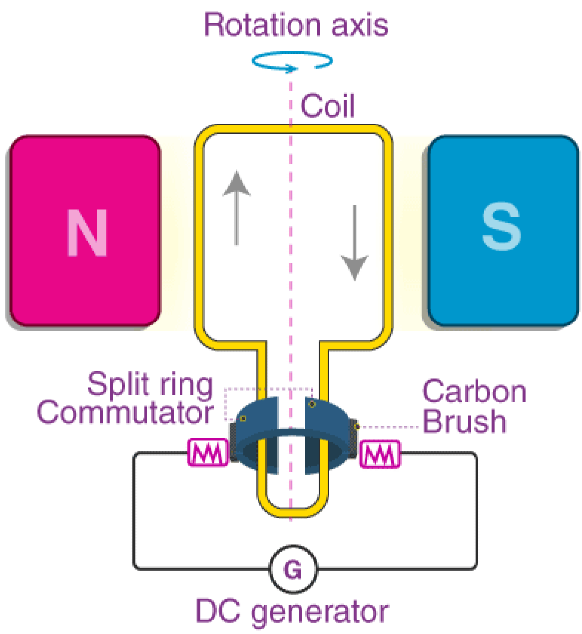


Figure 2. 3

- **Electric motors**
- **Galvanometers**
- **Moving-coil instruments**

Consider a Rectangular Current Loop

Let the loop have:

- length = l
- breadth = b
- area = $A = l \times b$
- current = I
- number of turns = N
- placed in a **uniform magnetic field** $\vec{B}$

Let the plane of the loop make an angle θ with the magnetic field.

Forces on the Loop

A magnetic force on a current-carrying conductor is:

$$\vec{F} = I\,\vec{L} \times \vec{B}$$

Important observations:

- The sides of length l are *parallel* to $\vec{B}$ → **no force**.
- The sides of length b are *perpendicular* to $\vec{B}$ → **maximum force**.

So we only consider the two vertical sides of the loop.

Magnitude of Force on Each Side

Force on one side:

$$F = IbB$$

These two forces are:

- Equal in magnitude

- Opposite in direction
- Separated by distance l

Hence they form a **couple** → produces **torque**

Derivation of Torque

Torque for a couple:

$$\tau = F \times \text{perpendicular distance}$$

Perpendicular distance between the two forces:

$$= l\sin\theta$$

Thus,

$$\tau = F(l\sin\theta)$$

Substitute $F = IbB$:

$$\tau = IbB(l\sin\theta)$$

But $lb = A$, the area of the loop.

So torque becomes:

$$\boxed{\tau = IAB\sin\theta}$$

For **N turns**:

$$\boxed{\tau = NIAB\sin\theta}$$

Maximum Torque

Torque is maximum when:

$$\theta = 90^\circ$$

Thus,

$$\tau_{\text{max}} = NIAB$$

Vector Form (Magnetic Dipole Moment)

Define the **magnetic dipole moment**:

$$\vec{m} = NIA\,\hat{n}$$

Then the torque becomes:

$$\boxed{\vec{\tau} = \vec{m} \times \vec{B}}$$

This is the most general expression.

Applications

- Operation of **DC motors**
- Principle of **moving-coil galvanometers**
- Torque on **magnetic dipoles**
- Electromechanical energy conversion

Moving Coil Galvanometer

A **galvanometer** is an instrument used to detect and measure **very small electric currents**. The most accurate and sensitive type is the **Moving Coil Galvanometer**, which works on the **principle of torque acting on a current-carrying coil placed in a magnetic field**. The torque produced by the magnetic field causes the coil to rotate, and this rotation is proportional to the current flowing through the coil. Hence, the scale of the instrument can be made uniform.

Principle

The galvanometer is based on:

"A current-carrying coil placed in a uniform magnetic field experiences a torque."

Torque:

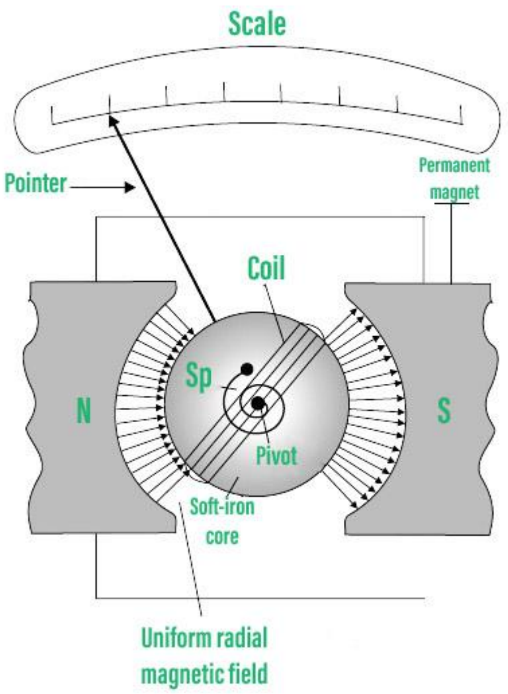
$$\tau = NIAB$$

where

- N = number of turns
- I = current

- A = area of the coil
- B = magnetic field strength

This torque causes angular deflection.



Construction of a Moving Coil Galvanometer

A standard **D'Arsonval / Weston type** moving coil galvanometer has the following parts:

(a) Permanent Magnet

- Strong **horse-shoe magnet**
- Provides a **radial magnetic field**
- Ensures coil experiences **maximum torque** at all angles
- Makes the galvanometer **linear** (scale uniform)

(b) Soft Iron Cylinder (Core)

- Placed between the poles of the magnet
- Concentrates magnetic field lines
- Reduces magnetic reluctance
- Ensures **uniform field**

- Increases sensitivity

(c) Coil

- A rectangular light coil of **thin copper wire**
- Wound on a **non-metallic frame** (usually aluminum)
- Number of turns = N
- Mounted on a pivot or suspension

(d) Suspension or Control System

Two types: *Radial type or pivot type*

1. **Upper Suspension**
    - Very thin **phosphor bronze strip**
    - Carries current into the coil
    - Provides restoring torque
2. **Lower Suspension**
    - A light spring or fibre
    - Carries current out
    - Maintains alignment of coil

Restoring torque:

$$\tau_{\text{restoring}} = k\theta$$

where

- k= torsional constant
- θ= deflection angle

(e) Mirror

- Small mirror attached to the suspension
- Light beam reflected on a scale

- Used for **accurate measurement**
- Common in laboratory galvanometers (tangent type)

(f) Pointer & Scale (in portable instruments)

- Pointer attached to coil
- Moves over a calibrated scale

Working of the Moving Coil Galvanometer

Let a current I flow through the coil.

Step 1: Magnetic Torque

In the magnetic field,

$$\tau_{\text{mag}} = NIAB$$

This torque tries to rotate the coil.

Step 2: Restoring Torque

As coil rotates, suspension wire twists and offers restoring torque:

$$\tau_{\text{restoring}} = k\theta$$

Step 3: Equilibrium

At steady deflection:

$$\tau_{\text{mag}} = \tau_{\text{restoring}}$$

$$NIAB = k\theta$$

Thus,

$$\theta = \frac{NAB}{k} I$$

Important Result

Deflection is directly proportional to current:

$$\theta \propto I$$

This gives a **uniform scale**, one of the major advantages of the MCG.

Sensitivity of a Galvanometer

Sensitivity = amount of deflection per unit current.

(a) Current Sensitivity

$$S_i = \frac{\theta}{I} = \frac{NAB}{k}$$

A galvanometer becomes highly sensitive if:

- Number of turns $N$is large
- Magnetic field $B$is strong
- Area $A$is large
- Torsional constant $k$is small

(b) Voltage Sensitivity

$$S_v = \frac{\theta}{V} = \frac{NAB}{kR}$$

where R= resistance of the galvanometer

Voltage sensitivity depends on both current sensitivity and coil resistance.

Galvanometer as Ammeter

- Ammeter measures **high current**
- A galvanometer measures only small current

  → So it is converted into ammeter by connecting a **low resistance (shunt) in parallel**

Galvanometer as Voltmeter

- Voltmeter measures **potential difference**

  → Convert galvanometer to voltmeter by connecting **high resistance in series**

Advantages

- High sensitivity
- Uniform scale
- Accurate and easy to read
- Very low power consumption
- Works well in DC circuits

Sources of Error

- Friction in pivot type instruments
- Aging of magnets (loss of strength)
- Temperature changes affecting wire resistance
- Mechanical vibration
- Dust or moisture in coil

Applications

- Detecting very small currents
- Laboratory experiments
- Basis for ammeters & voltmeters
- Used in measuring bridges & null point methods

Damping Correction

When a moving coil galvanometer receives a small current, the coil rotates and reaches a new equilibrium position.

However, the coil **does not stop instantly** at this final position.

Instead, it:

- overshoots the final deflection,
- oscillates back and forth,
- and slowly comes to rest.

This oscillatory behaviour is due to the **inertia of the moving coil system**.

To make the pointer settle quickly and accurately, **damping mechanisms** are provided.

Types of Damping

There are **three** types of damping in galvanometers:

(a) Air Damping

- A vane (aluminum/iron) attached to the coil moves inside an air chamber.
- The air resistance opposes motion.
- Reduces oscillations.

(b) Electromagnetic Damping (main damping)

- Aluminum frame of the coil cuts the magnetic field while moving.
- This induces **eddy currents**.
- These eddy currents produce an opposing torque (Lenz's Law).
- Strong, smooth, stable damping.

(c) Controlling (Restoring) Torque

- Produced by suspension/wire or springs.
- Helps coil return to zero position.
- Indirectly contributes to damping.

The Need for Damping Correction

When we want *very accurate measurements*, the damping effects slightly shift the equilibrium position of the coil.

This means:

- The actual reading

- is **slightly different**
- from the *ideal* deflection (without damping).

Thus, a **damping correction** is needed to find the true value of current.

This correction becomes important when:

- measuring small currents,
- studying galvanometer constant,
- doing ballistic galvanometer experiments,
- measuring charge (q = kθ),
- measuring high precision null point readings.

Damping Correction – Theory

Let

- θ_0= true (ideal) deflection without damping
- θ= observed deflection with damping
- λ= damping factor (logarithmic decrement)

In the presence of damping, the coil loses some energy each oscillation.

The damping reduces the deflection roughly by a factor proportional to λ.

For small damping, the correction is:

$$\theta_0 = \theta(1 + \frac{\lambda}{2})$$

This is the **damping correction formula**.

Alternatively (more exact):

$$\theta_0 = \theta(1 + \frac{\lambda^2}{16})$$

But for most galvanometer calculations, the first approximation is enough.

How λ (logarithmic decrement) is measured

Let

- θ_1= 1st maximum deflection
- θ_2= 2nd maximum deflection on the same side

Then:

$$\lambda = \ln(\frac{\theta_1}{\theta_2})$$

This value is substituted in damping correction formulas.

Why Damping Correction is Necessary

- To get the **exact value** of current or charge.
- Without correction, the reading is **smaller** than the true reading.
- This is important when the galvanometer is used as:
    - a **ballistic galvanometer**
    - a **standard instrument**
    - for **calibration**
    - for **research level experiments**

Ampere's Circuital law

Ampère's Circuital Law is one of the **fundamental laws of magnetism**.

It relates the **magnetic field** around a closed path to the **total current** passing through the surface bounded by that path.

This law is the magnetic equivalent of:

- **Gauss's Law** (electric field)
- **Kirchhoff's current law** (current flow)

It is a crucial part of **Maxwell's equations** and helps determine the magnetic field for symmetric current distributions.

Statement of the Law

"The line integral of the magnetic field around any closed path is equal to μ_0 times the total current enclosed by the path."

Mathematically:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

Where

- $\oint$ → closed loop integral
- $\vec{B}$→ magnetic field
- $d\vec{l}$→ small element of the closed path
- I_{enc}→ total current enclosed
- μ_0→ permeability of free space $4\pi \times 10^{-7}$ H/m

Physical Meaning

The law tells us:

- Wherever there is **current**, there is **magnetic field**.
- The magnetic field forms **closed loops** around the current.
- The **strength** of this magnetic field depends on the **current enclosed** by the chosen path.

Ampère’s law provides a direct method to calculate **B**, especially for symmetric situations.

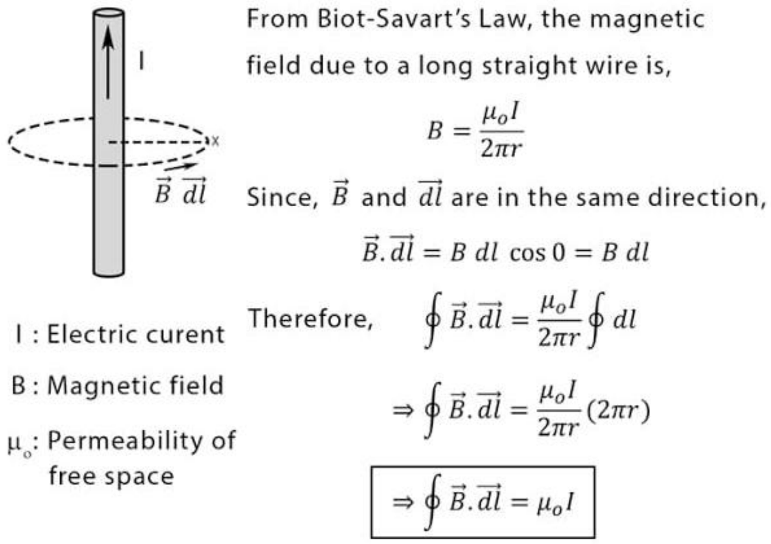


Figure 2. 4

Ampère’s law is derived from the **Biot–Savart law**, which gives the magnetic field due to a current element.

For a long straight conductor, the magnetic field at distance r is:

$$B = \frac{\mu_0 I}{2\pi r}$$

If we take a circular Amperian loop of radius r:

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r)$$

Substitute B:

$$\oint \vec{B} \cdot d\vec{l} = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

This directly matches Ampère’s law.

Conditions for Using Ampère's Law

Ampère's law is especially powerful when the magnetic field has:

- **Symmetry (cylindrical, planar, toroidal)**
- **Constant magnitude** over the Amperian loop
- **Direction parallel/anti-parallel** to the chosen path

Examples where Ampère's law is ideal:

- Long straight wire
- Solenoid
- Toroid
- Coaxial cable
- Infinite sheet of current

Applications

1. Magnetic field due to a long straight conductor

Using Ampère's law:

$$B = \frac{\mu_0 I}{2\pi r}$$

2. Magnetic field inside a solenoid

For a solenoid with n turns/m:

$$B = \mu_0 n I$$

3. Magnetic field inside a toroid

For a toroidal coil of mean radius R:

$$B = \frac{\mu_0 N I}{2\pi R}$$

4. Coaxial cable

Field in different regions can be obtained very easily using Ampère's law.

Differential Form (Maxwell's Form)

To obtain the differential form of Ampère's law starting from its integral form.

Integral form:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

Differential form:

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

Ampère's Circuital Law (Integral Form)

Ampère's law states that the line integral of magnetic field around any closed loop equals μ_0 times the current enclosed by that loop:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

The enclosed current is written using current density $\vec{J}$:

$$I_{\text{enc}} = \iint_S \vec{J} \cdot d\vec{S}$$

Substituting:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \iint_S \vec{J} \cdot d\vec{S}$$

Use of Stokes' Theorem

Stokes' theorem relates a surface integral to a line integral:

$$\oint_{\partial S} \vec{A} \cdot d\vec{l} = \iint_S (\nabla \times \vec{A}) \cdot d\vec{S}$$

Let $\vec{A} = \vec{B}$. Then:

$$\oint \vec{B} \cdot d\vec{l} = \iint_S (\nabla \times \vec{B}) \cdot d\vec{S}$$

Substitute Stokes' Theorem into Ampère's Law

Replace the left-hand side of Ampère's law:

$$\iint_S (\nabla \times \vec{B}) \cdot d\vec{S} = \mu_0 \iint_S \vec{J} \cdot d\vec{S}$$

Since both integrals are surface integrals over the same surface:

$$(\nabla \times \vec{B}) = \mu_0 \vec{J}$$

This equality must hold at every point on the surface.

Final Differential Form

$$\boxed{\nabla \times \vec{B} = \mu_0 \vec{J}}$$

Limitations

Ampère's Law originally failed for:

- Time-varying electric fields
- Open circuits (like capacitor charging)

Maxwell corrected this by introducing the **displacement current**.

Divergence of magnetic field

In classical electromagnetism, the concept of **divergence** is extremely important because it helps us understand whether a field originates from or converges into a point (like a source or sink).

For electric fields, charges act as sources and sinks; hence, the divergence of the electric field is related to charge density (Gauss's law for electricity).

But when we study **magnetic fields**, experiments tell us something fundamentally different: no isolated magnetic charges (monopoles) have ever been detected. Every magnet we know has both a north and south pole. Even if you break a magnet into smaller pieces, each piece still has both poles.

This leads to a foundational law of electromagnetism:

Magnetic field lines always form closed loops and never begin or end at any point.

This property is mathematically expressed using divergence.

Statement

The divergence of the magnetic field at any point in space is always zero.

Mathematically,

$\nabla \cdot \mathbf{B} = 0$

This is known as **Gauss's law for magnetism**, one of Maxwell's equations.

1. Magnetic field lines have **no starting or ending points**.
2. There are **no magnetic monopoles** in nature.
3. Magnetic field lines form **continuous closed loops**.
4. At no point does the magnetic field "spread out" from a source like electric fields do from charges.

Thus, **the magnetic field is solenoidal** — meaning its field lines are continuous.

We use Gauss's divergence theorem to derive the differential form.

Step 1: Integral Form of Gauss's Law for Magnetism

Experimentally, we find that the **net magnetic flux through any closed surface is zero**.

$\oint \mathbf{B} \cdot d\mathbf{A} = 0$

This means that as many magnetic field lines enter a closed surface as leave it.

There is **no net outward flow** of magnetic field.

Step 2: Applying Gauss's Divergence Theorem

Gauss's theorem states:

$$\oint \mathbf{B} \cdot d\mathbf{A} = \iiint (\nabla \cdot \mathbf{B})\, dV$$

Since experiments show

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

we substitute this into Gauss's theorem:

$$\iiint (\nabla \cdot \mathbf{B})\, dV = 0$$

But this must be true for **any arbitrary volume**.

The only way this can be universally true is:

$$\nabla \cdot \mathbf{B} = 0$$

This is the **differential form** of Gauss's law for magnetism.

Interpretation (Conceptual Understanding)

1. Divergence tells us about **net outflow** of a field from a point.
2. Since $\nabla \cdot B = 0$, magnetic fields never emanate from a point.
3. Therefore, there are **no magnetic charges** (monopoles).
4. Every magnetic field line that enters a region must leave the region.
5. Magnetic fields are **continuously circulating** fields.

Importance in Electromagnetism

1. Forms one of the **four Maxwell's equations**.

2. Helps enforce the idea that magnetic fields are purely the result of
   - moving charges (currents)
   - changing electric fields
3. Essential for describing electromagnetic waves, inductors, and magnetic materials.
4. Explains why breaking a magnet never isolates a single pole.

Magnetic induction due to toroid

A **toroid** is a ring-shaped coil obtained by bending a long solenoid into a circular form. It consists of closely wound turns of wire on a doughnut-shaped core (usually air, iron, or ferrite).

A toroid is extremely useful in electromagnetism because:

1. It confines the magnetic field completely within its core.
2. The magnetic field outside a toroid is almost zero.
3. It avoids flux leakage, which is common in solenoids.
4. It forms the basis for transformers, inductors, magnetic memory devices, and energy storage coils.

A toroid acts like a **solenoid bent into a circle**, so the magnetic field inside is strong and uniform along the circular path.

Physical Idea behind Magnetic Field in a Toroid

When a current **I** flows through the N tightly wound turns of the toroid, each turn produces a magnetic field.

Because the coil is wound symmetrically in a circle, these fields add up **only inside the toroid** and cancel out elsewhere.

When current I flows through each of the N turns, each turn contributes a magnetic field inside the core.

Because the coil is circular and symmetric, the magnetic fields add up **inside** and cancel

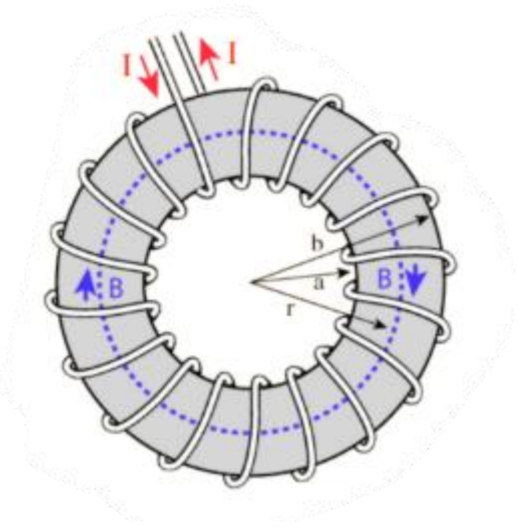


Figure 2. 5

outside.

Thus:

Inside the toroid → strong magnetic field

Outside the toroid → nearly zero

Central cavity → zero field

Applying Ampere's Circuital Law

Ampere's law states:

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enclosed}}$$

Choose an Amperian loop of radius r inside the toroid.

Because **B** is tangential and constant along this circular path,

$$\oint \mathbf{B} \cdot d\mathbf{l} = B(2\pi r)$$

Ampere's law becomes:

$$B(2\pi r) = \mu_0 I_{\text{enclosed}}$$

Current Enclosed

There are Nturns, each carrying current I.

$$I_{\text{enclosed}} = NI$$

Substitute into Ampere's law:

$$B(2\pi r) = \mu_0 NI$$

Therefore,

$$\boxed{B = \frac{\mu_0 NI}{2\pi r}}$$

This is the magnetic induction at a distance $r$from the center of the toroid.

Using Turns per Unit Length

Define:

$$n = \frac{N}{2\pi r}$$

Then the magnetic field becomes:

$$\boxed{B = \mu_0 nI}$$

Why Does $B \propto \frac{1}{r}$?

As you move outward, the circular path increases in length $(2\pi r)$.

The same total enclosed current spreads over a larger loop, so the field decreases with r.

Field Outside the Toroid

Take an Amperian loop outside the toroid.

It encloses equal numbers of turns that contribute currents in opposite directions (net effect zero):

$$I_{\text{enclosed}} = 0$$

Hence,

$$B = 0$$

Field in the Central Cavity

Take an Amperian loop inside the hollow center.

No current passes through:

$$I_{\text{enclosed}} = 0$$

Therefore,

$$B = 0$$

Toroid with Magnetic Core

If the toroid has a core of relative permeability μ_r,

$$\boxed{B = \frac{\mu_0 \mu_r N I}{2\pi r}}$$

Characteristics of Magnetic Field in a Toroid

1. Confined within the core.
2. Circular and tangential.
3. Varies as $\frac{1}{r}$.
4. Stronger at inner radius, weaker at outer radius.
5. Almost zero external field.

Applications

1. Inductors
2. Transformers
3. Toroidal magnets
4. Magnetic confinement systems (Tokamak)
5. EMI filters
6. Ferrite cores

Unit 3: Magnetism and Electro-Magnetic Induction

1. Magnetic Induction B
2. Magnetization M
3. Relation between B, H and M
4. Magnetic Susceptibility
5. Magnetic Permeability
6. Experiment to draw B-H Curve
7. Energy loss due to hysteresis
8. Importance of hysteresis curve
9. Faradayand Lenz laws (Vector form)
10. Coefficient of self inductoance of solenoid
11. Anderson's Method
12. Mutual Inductance
13. Coefficient of Mutual Inductance between two Coaxial Solenoids
14. Coefficient of coupling

Magnetic Induction B

Magnetic induction, commonly represented by the symbol **B**, is one of the most fundamental quantities in magnetism. It tells **how strong** the magnetic field is and **how it acts** on moving charges or current-carrying conductors. In simple terms, the magnetic induction B describes the **magnetic influence** present in a region of space.

B is also called:

• Magnetic field

• Magnetic flux density

• Magnetic induction vector

The magnetic induction vector **B**describes:

1. The **strength** of the magnetic field.
2. The **direction** of the field at every point in space.
3. The **force** experienced by moving charges or current-carrying wires.

Think of it like:

How gravity tells you how strongly objects are pulled down,

B tells you how strongly charges are pushed sideways due to magnetism.

The magnetic induction B is defined using the **Lorentz force**:

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

Where:

F= magnetic force

q= charge

v= velocity of the charge

B= magnetic induction

From this, magnitude of B is defined as:

$$B = \frac{F}{qv\sin\theta}$$

θ = angle between velocity vector and magnetic field.

This gives a very physical meaning:

B is the magnetic field that produces **1 newton of force** on a **1-coulomb charge** moving at **1 m/s** perpendicular to the field.

A straight conductor of length L carrying current I in a magnetic field experiences a force:

$$\mathbf{F} = I(\mathbf{L} \times \mathbf{B})$$

Magnitude:

$$F = BIL\sin\theta$$

Thus,

$$B = \frac{F}{IL\sin\theta}$$

This is extremely practical since most experiments use current instead of isolated charges.

The SI unit of B is the **tesla (T)**.

Definition:

A magnetic field of 1 tesla exerts a force of 1 newton per meter on a wire carrying 1 ampere of current.

$$1\ \mathrm{T} = 1\ \frac{\mathrm{N}}{\mathrm{A\backslash cdotpm}}$$

Smaller units often used:

• 1 millitesla (mT)

• 1 microtesla (μT)

• 1 gauss (G), where 1 G = 10^{-4} T

Magnetic induction B is related to magnetic flux Φ through the area A:

$$\Phi = \int \mathbf{B} \cdot d\mathbf{A}$$

If B is uniform and perpendicular to area:

$$\Phi = BA$$

This relation is fundamental in electromagnetic induction.

Magnetic induction is represented through **magnetic field lines**:

1. Density of lines → strength of B
2. Direction of lines → direction of B
3. Lines never start or end; they form continuous loops
4. Closer spacing → stronger field

This also supports Maxwell's equation:

$$\nabla \cdot \mathbf{B} = 0$$

Meaning:

Magnetic monopoles don't exist; field lines always loop back.

Magnetization M

When a magnetic material (like iron, steel, cobalt, etc.) is placed inside a magnetic field, the atoms inside it respond. Each atom behaves like a tiny magnetic dipole due to electron spin and orbital motion. When these tiny dipoles align in some direction, the material becomes magnetized. The **measure of how strongly a material becomes magnetized** in response to an external magnetic field is called **Magnetization**, denoted by **M**. It tells us how many magnetic dipoles are produced inside the material and how strongly they are oriented.

Definition of Magnetization

Magnetization M is defined as the **magnetic dipole moment per unit volume** of the material.

$$M = \frac{\text{Magnetic dipole moment}}{\text{Volume}}$$

If N magnetic dipoles, each of moment m, occupy volume V:

$$M = \frac{Nm}{V}$$

Thus, M represents the **average dipole alignment** inside the material.

Physical Meaning

Magnetization tells you:

1. How strongly dipoles inside the material are aligned
2. Whether the material supports or resists magnetism
3. How the material contributes to the total magnetic field inside it

High M → strongly magnetized

Low M → weakly magnetized

Zero M → unmagnetized

Units of Magnetization

$$\text{SI unit of } M = \text{A/m}$$

It is not expressed in tesla because M measures dipole density, not field strength.

Relation Between B, H, and M

In a magnetic material, the total magnetic field B comes from two sources:

1. External applied magnetic field (H)
2. Magnetization of material (M)

The relation is:

$$B = \mu_0(H + M)$$

Where

B= magnetic induction

H= magnetizing field

M= magnetization

μ_0= permeability of free space

This equation shows how magnetization contributes directly to the magnetic field inside a material.

Magnetic Intensity (H) vs Magnetization (M)

H is the field applied **from outside**.

M is the field generated **inside the material** because of dipole alignment.

External cause → H

Internal response → M

Together they produce the total magnetic field B.

Magnetization Curve

When a material is subjected to increasing H, the magnetization M increases but not linearly forever.

Stages:

1. Initial region: M rises quickly
2. Near saturation: M increases slowly
3. Saturation: M becomes almost constant

This is why ferromagnetic materials saturate after a point.

Magnetization in Different Materials

1. Diamagnetic Materials

Magnetization is weak and opposite to applied field.

$$M < 0$$

Examples: Copper, silver, water.

2. Paramagnetic Materials

Magnetization is weak and in same direction as field.

$$M > 0 \text{ (small)}$$

Examples: Aluminium, oxygen.

3. Ferromagnetic Materials

Strong magnetization even for small fields.

$$M \gg H$$

Examples: Iron, cobalt, nickel.

These materials have domain structure where dipoles strongly align.

Magnetic Susceptibility

Magnetization depends on applied field H:

$$M = \chi_m H$$

Where χ_m= magnetic susceptibility.

Diamagnetism → $\chi_m < 0$

Paramagnetism → $\chi_m > 0$

Ferromagnetism → $\chi_m \gg 1$

This tells how easily a material can be magnetized.

Importance of Magnetization

1. It determines behavior of magnetic materials.
2. Essential in transformer cores, motors, and inductors.
3. Explains permanent magnets.
4. Appears in the Maxwell equation for magnetic materials.

5. Governs magnetic energy storage.

Basically, M gives the **material's internal magnetic character**.

Summary

Magnetization M:

• measures magnetic dipole moment per unit volume

• has SI unit A/m

• determines the internal magnetic response of materials

• is linked with total field by $B = \mu_0(H + M)$

• varies depending on material type (dia, para, ferro)

• plays a major role in real-life magnetic devices

Relation between B, H and M

In magnetic materials, the magnetic field inside the substance is not determined only by the externally applied magnetic field. It also depends on how the material responds to that external field. When a magnetic field is applied, the atoms or magnetic dipoles inside the material get aligned, producing an additional internal magnetic field.

Thus, the total magnetic effect inside a material is influenced by three fundamental quantities:

1. Magnetic Field Intensity (H) – the external magnetising field
2. Magnetisation (M) – magnetic dipole moment per unit volume induced in the material
3. Magnetic Flux Density (B) – the total magnetic field inside the material

Understanding the relationship between these quantities helps explain the magnetic behaviour of different materials such as diamagnetic, paramagnetic, and ferromagnetic substances.

1. Magnetic Field Intensity (H)

Magnetic field intensity (H) represents the external magnetic field applied to a material.

It is the measure of the magnetising force, independent of the material medium.

- Symbol: H
- SI Unit: A/m
- Physical Meaning: Determines how strongly we are attempting to magnetise a material.

2. Magnetisation (M)

Magnetisation (M) is the magnetic dipole moment per unit volume of the material.

It represents the extent to which the material becomes magnetised when subjected to the external field.

- Symbol: M
- SI Unit: A/m
- Physical Meaning: Shows the contribution of the material's internal magnetic structure to magnetism.

When H is applied, the magnetic domains align, and magnetisation increases.

3. Magnetic Flux Density (B)

Magnetic flux density (B) is the total magnetic field inside the material.

It includes both the applied field and the internal field due to magnetisation.

- Symbol: B
- SI Unit: Tesla (T)
- Physical Meaning: Represents the actual magnetic field present in the medium.

Fundamental Relation Between B, H and M

The total magnetic field inside a material is given by

$B = \mu_0(H + M)$

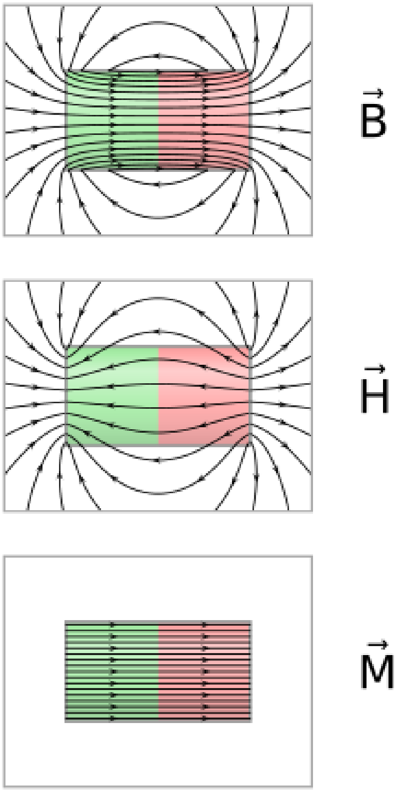


Figure 3. 1

where

B= magnetic flux density

H= magnetic field intensity

M= magnetisation

$\mu_0 = 4\pi \times 10^{-7}$ H/m(permeability of free space)

This equation shows that

- Hrepresents the applied field
- Mrepresents the induced field
- Together, they produce B, the total field inside the material

Using Magnetic Susceptibility (χ)

Magnetisation is related to H by magnetic susceptibility:

$$M = \chi H$$

Substituting in the earlier equation:

$$B = \mu_0(H + \chi H)$$

$$B = \mu_0(1 + \chi)H$$

Define relative permeability:

$$\mu_r = 1 + \chi$$

Thus,

$$B = \mu_0 \mu_r H = \mu H$$

where $\mu = \mu_0 \mu_r$ is the absolute permeability of the material.

Summary Table

Quantity	Symbol	Definition	Unit
Magnetic field intensity	H	Applied magnetic field	A/m
Magnetisation	M	Magnetic dipole moment per unit volume	A/m
Magnetic flux density	B	Total field inside material	Tesla
Magnetic susceptibility	χ	Ability of material to get magnetised	Dimensionless
Permeability	μ	Ability to permit magnetic field	H/m

Magnetic Susceptibility

When a magnetic material is placed in an external magnetic field, its atoms or molecules try to align with the applied field. This alignment produces an additional internal magnetic moment within the material, known as **magnetisation (M)**.

Different materials respond differently to the same applied magnetic field.

Some materials are weakly magnetised, some strongly magnetised, and some even oppose the applied field.

To measure how easily or how strongly a material becomes magnetised due to an external field, we define a physical quantity called **magnetic susceptibility**, denoted by **χ**.

Magnetic susceptibility (χ) is defined as:

The ratio of magnetisation (M) produced in a material to the magnetic field intensity (H) applied to the material.

Mathematically,

$$\chi = \frac{M}{H}$$

or,

$$M = \chi H$$

Magnetic susceptibility is a dimensionless quantity.

- Magnetisation M represents the magnetic dipole moment per unit volume developed inside a material.
- Magnetic field intensity H represents the external magnetic field applied.
- Susceptibility χ tells how much magnetisation is produced for a given applied field.

Thus, susceptibility indicates **the degree of magnetisation of a material in response to an external magnetic field**.

Physical Interpretation

1. If a material has **high susceptibility**, it becomes **strongly magnetised** even under a small applied field.
2. If a material has **low susceptibility**, it becomes **weakly magnetised**.
3. If susceptibility is **negative**, the magnetisation is in the **opposite direction** to the applied field.

Thus, susceptibility gives information about:

- the nature of the material,
- the strength of magnetisation,
- whether the material is attracted or repelled by a magnetic field.

Types of Magnetic Susceptibility Based on Materials

Magnetic susceptibility varies depending on the type of material:

1. Diamagnetic Materials

- χ is **negative** and very small

$$\chi \approx -10^{-5} \text{ to } -10^{-6}$$

- These materials develop magnetisation in the **opposite direction** to the applied field.
- They are weakly repelled by magnetic fields.

Examples: Copper, Silver, Gold, Bismuth

2. Paramagnetic Materials

- χ is **positive** and small

$$\chi \approx +10^{-5} \text{ to } +10^{-3}$$

- They develop magnetisation in the **same direction** as the applied field.
- They are weakly attracted by magnetic fields.

Examples: Aluminium, Platinum, Sodium

3. Ferromagnetic Materials

- χ is **very large and positive**

$$\chi \gg 1$$

- They become **strongly magnetised** even under a small applied field.
- Magnetisation persists even after removing the external field.

Examples: Iron, Cobalt, Nickel

Relation with Permeability

Magnetic susceptibility is related to a material's relative permeability μ_r:

$$\mu_r = 1 + \chi$$

Thus, permeability of a material (how easily magnetic field can pass through it) depends directly on susceptibility.

Higher susceptibility → higher permeability.

Factors Affecting Magnetic Susceptibility

1. **Temperature**
   - For paramagnetic materials:

$$\chi \propto \frac{1}{T} \text{(Curie's Law)}$$

   - For ferromagnetic materials: susceptibility decreases as temperature increases.
2. **Nature of the material**
   - Atomic structure determines how easily dipoles align.
3. **Impurities**
   - Presence of other elements can increase or decrease susceptibility.

Significance of Magnetic Susceptibility

Magnetic susceptibility helps determine:

- how a material behaves in a magnetic field,
- whether magnetisation aligns or opposes the applied field,
- the classification of materials,
- design of magnetic devices like transformers, cores, inductors, memory devices, and sensors.

It is a fundamental property used in physics, chemistry, geophysics, and material science.

Magnetic Permeability

Magnetic permeability is a fundamental property of materials that describes **how easily magnetic field lines can pass through the material**.

In other words, permeability tells us **how well a material supports the formation of a magnetic field inside it**.

It is one of the key parameters in understanding magnetic circuits, electromagnets, transformers, inductors, and all magnetic materials.

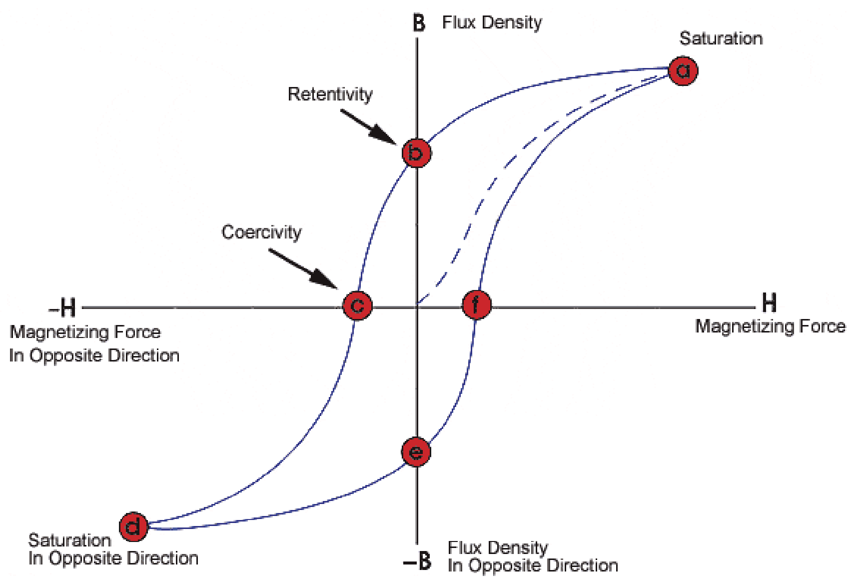


Figure 3. 2

Definition

Magnetic permeability (μ) is defined as:

$$\mu = \frac{B}{H}$$

where

B = Magnetic flux density (Tesla)

H = Magnetic field intensity (A/m)

Thus, permeability tells us how much magnetic flux density is produced inside a material when a magnetic field is applied.

Unit: Henry per meter (H/m)

Types of Permeability

1. Absolute Permeability (μ)

The actual permeability of any material.

$$B = \mu H$$

2. Permeability of Free Space (μ_0)

Also called **vacuum permeability**.

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

This is a constant value.

3. Relative Permeability (μ_r)

It compares the permeability of a material to free space.

$$\mu_r = \frac{\mu}{\mu_0}$$

No unit (dimensionless).

Physical Meaning

- If a material has **high permeability**, magnetic field lines easily pass through it.
- If a material has **low permeability**, magnetic field lines pass with difficulty.

- If permeability is **very high** (like in iron), the material becomes **strongly magnetised** even with small magnetic fields.

Thus, permeability indicates the **ease of magnetisation**.

Relation With Susceptibility

Magnetic susceptibility (χ) measures how easily a material gets magnetised.

Permeability is related to susceptibility as:

$$\mu = \mu_0(1 + \chi)$$

And

$$\mu_r = 1 + \chi$$

So,

• When χ is high → material has high permeability

• When χ is low → permeability is low

Permeability in Different Materials

1. Diamagnetic Materials

- $\mu_r < 1$
- Permeability slightly less than free space
- Weakly repelled by magnetic fields

  Examples: Copper, Bismuth

2. Paramagnetic Materials

- μ_r slightly > 1
- Weakly attracted

  Examples: Aluminium, Sodium

3. Ferromagnetic Materials

- $\mu_r >> 1$ (hundreds to thousands)
- Very strongly magnetised

  Examples: Iron, Nickel, Cobalt

Ferromagnetic materials are used in electromagnets and transformer cores because of their extremely high permeability.

Practical Importance

Magnetic permeability plays a key role in:

- Electromagnets
- Transformers
- Inductors
- Magnetic shielding
- Motors and generators
- Magnetic storage devices

Materials with high permeability concentrate magnetic flux and make devices more efficient.

Experiment to draw B-H Curve

A **B–H curve** (Magnetisation Curve) shows how a magnetic material responds when an external magnetising field **H** is applied.

It helps us study:

- Magnetisation behaviour
- Saturation
- Retentivity
- Coercivity

- Hysteresis

To obtain this curve experimentally, a **toroidal ring specimen** with coils wound on it is used.

To **plot the B–H curve** of a ferromagnetic specimen by measuring:

- **Magnetic field intensity (H)** using the magnetising coil
- **Magnetic flux density (B)** using the search coil and ballistic galvanometer

Apparatus Required

- Toroidal iron ring specimen
- Magnetising coil (primary coil)
- Search coil (secondary coil)
- Ballistic galvanometer
- Rheostat
- Ammeter
- Voltmeter
- Keys and connecting wires
- DC regulated supply

Circuit Diagram

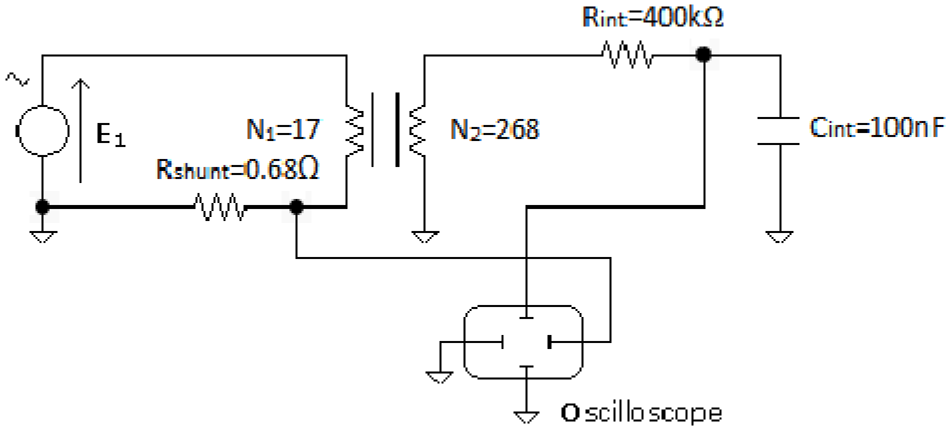


Figure 3. 3

Theory

For a toroidal core:

Magnetising Field (H)

$$H = \frac{NI}{l}$$

where

N = number of turns in magnetising coil

I = current in magnetising coil

l = mean magnetic path length of the toroid

Magnetic Flux Density (B)

Flux change induces a galvanometer deflection:

$$B = k\,\theta$$

where

θ = galvanometer deflection

k = calibration constant for search coil + galvanometer system

Thus, for different values of **I**, you find corresponding **H** and **B**, and plot **B vs H**.

Procedure

1. Initial Setup

- Connect the magnetising coil in series with ammeter, rheostat, DC supply.
- Connect the search coil to the ballistic galvanometer.
- Ensure connections are tight and polarity correct.

2. Increasing Magnetising Current

- Slowly increase the current in steps using the rheostat.

- At each current value:
  - Note the ammeter reading (to calculate **H**)
  - Momentarily press the search coil key → galvanometer deflects
  - Record the deflection (to calculate **B**)

3. Obtaining the B–H Curve

- Continue increasing current until the material **reaches saturation**.
- Plot B against H → gives the **magnetisation curve**.

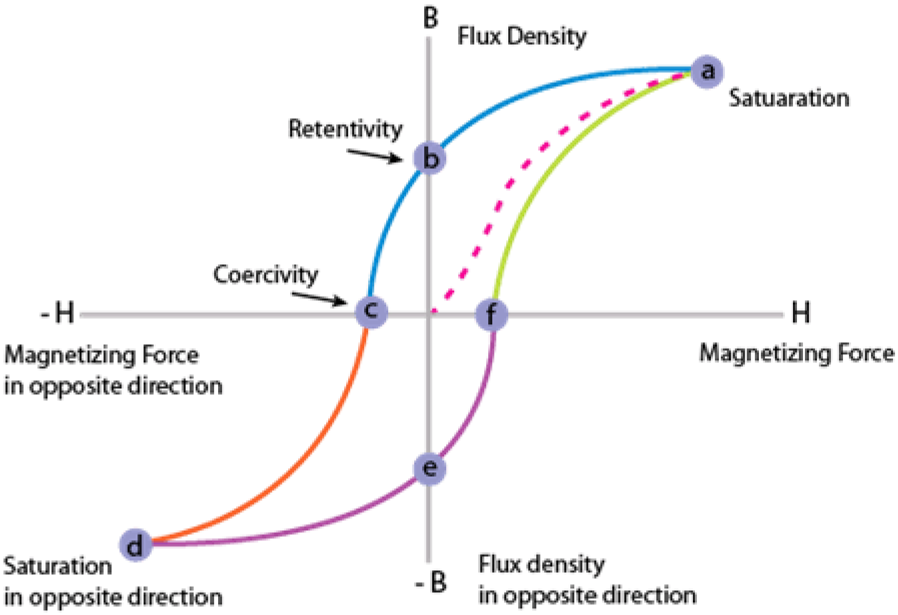


Figure 3. 4

4. Obtaining the Hysteresis Loop

- After saturation, gradually **reduce** the current back to zero.
- Now make current flow in the **reverse direction**.
- Note deflection values during decreasing and reversing of current.
- Continue until saturation in the opposite direction is reached.
- Plot entire cycle — this gives the **hysteresis loop**.

The curve shows:

- **O–A:** Initial magnetisation

- **A–B:** Saturation
- **C:** Residual magnetism (Retentivity)
- **D:** Coercive force (Coercivity)

Observations:

Current (I)	Magnetising Field (H)	Deflection (θ)	Flux Density (B)
...	...	...	...

Result

1. **The B–H curve** of the given ferromagnetic specimen was successfully plotted.
2. The material shows **saturation**, **retentivity**, and **coercivity**, typical of ferromagnets.
3. From the hysteresis loop, energy loss per cycle can also be estimated.

Energy loss due to hysteresis

When a magnetic material (like iron) is taken through a complete cycle of magnetization — from zero field to positive saturation, back through zero, then to negative saturation, and again to zero — the **magnetic induction (B)** does **not** follow the same path as the magnetizing field **H**.

This lag between **B** and **H** forms a closed loop called the **hysteresis loop**.

Because of this lag, some energy is **lost** in every cycle of magnetization. This loss appears as **heat** inside the magnetic material.

This is known as **hysteresis loss**.

Why Does Energy Get Lost?

A magnetic material consists of many tiny magnetic domains.

During magnetization:

- Domains **rotate**
- Domain walls **move**
- Some domains are **reluctant to align** with the external field
- Internal friction inside the material **resists domain motion**

Every time H changes direction, the domains must rearrange again.

This rearrangement is not perfectly reversible — it **consumes energy**.

This consumed energy becomes **heat** inside the core.

The energy lost per cycle of magnetization is equal to the area enclosed by the hysteresis loop on the B–H curve.

Mathematically,

$$\text{Energy loss per unit volume per cycle} = \oint H\, dB$$

This integral represents the **area of the hysteresis loop**.

How the Energy is Manifested

- The core gets **hot** during AC operation
- Heat must be removed to prevent damage
- Transformers and motors use materials with **narrow hysteresis loops** to reduce heating

Hysteresis Loss Formula

If an AC magnetic field of frequency f cycles per second is applied, the total loss per second is:

$$P_h = \eta\, B_{max}^{1.6}\, f\, V$$

Where:

- P_h= hysteresis power loss
- η= Steinmetz constant (depends on the material)
- B_{max}= maximum flux density
- f= frequency
- V= volume of the material

Key Points

- **Hysteresis loss = heat generated due to repeated magnetization and demagnetization.**
- It depends on **area of hysteresis loop**.
- Soft iron has **narrow loop → low hysteresis loss** (used in transformer cores).
- Hard steel has **wide loop → high hysteresis loss** (used in permanent magnets).
- Loss increases with **frequency** and **maximum flux density**.

Importance of hysteresis curve

The **hysteresis curve**, also called the **B–H loop**, shows how a magnetic material responds when it is magnetized and demagnetized.

It reveals the **magnetic characteristics** of the material, such as:

- How easily it can be magnetized
- How much magnetic flux it can carry
- How much energy it loses as heat
- Whether it is suitable for transformers, motors, or permanent magnets

Because of this, the hysteresis loop is basically the "fingerprint" of a magnetic material.

1. Determines Hysteresis Loss

The **area of the loop** gives the **energy lost per cycle** of magnetization.

- **Narrow loop → low loss** (good for transformers, AC machines)
- **Wide loop → high loss** (good for permanent magnets)

This helps engineers select the right material for minimizing heat generation.

2. Shows Retentivity (Residual Magnetism)

Retentivity = the **value of B** left in the material when **H becomes zero**.

This tells us how strongly the material can remain magnetized.

- High retentivity → useful for **permanent magnets**
- Low retentivity → useful for **transformer cores** (they should not retain magnetism)

3. Shows Coercivity (Required Reverse Field)

Coercivity = the **reverse magnetizing force** needed to bring flux density back to zero.

- High coercivity → **hard magnetic materials** (steel)
- Low coercivity → **soft magnetic materials** (soft iron, silicon steel)

This helps choose materials for switches, relays, magnets, etc.

4. Distinguishes Between Soft and Hard Magnetic Materials

Soft magnetic materials:

- Narrow hysteresis loop
- Low coercivity
- Low retentivity
- Low hysteresis loss

Hard magnetic materials:

- Wide hysteresis loop
- High coercivity
- High retentivity

- Used for permanent magnets

5. Helps in Designing Transformer and Motor Cores

To reduce heating in AC machines, materials **must** have:

- Narrow hysteresis loop
- High permeability
- Low coercivity

The B–H curve helps engineers confirm these properties.

6. Useful for Selecting Materials for Specific Applications

Application	Material Property Needed	Loop Type
Transformers	Low hysteresis loss	Narrow loop
Permanent magnets	High retentivity	Wide loop
Electromagnets	High permeability	Steep initial slope
Magnetic recording devices	High coercivity	Wide loop

7. Shows Saturation Level

The top part of the curve shows when the material is **saturated**, meaning it cannot be magnetized further.

This is crucial while designing transformers and inductors so they don't enter saturation.

Conclusion

The hysteresis curve is important because it:

- Reveals complete magnetic behavior
- Helps choose the correct magnetic material
- Predicts energy loss and heating
- Shows coercivity, retentivity, and saturation
- Decides applications like transformers, motors, and magnets

Faraday and Lenz laws (Vector form)

When magnetic flux linked with a circuit changes, an **emf (electromotive force)** is induced.

Faraday discovered **how much emf** is produced.

Lenz explained **the direction** of that emf.

Together, they form the core of electromagnetic induction — the principle behind generators, transformers, induction stoves, and more.

1. Faraday's Law of Electromagnetic Induction (Vector Form)

Faraday's law states:

The induced emf in a closed loop equals the negative rate of change of magnetic flux through the loop.

Vector Form

$$\mathcal{E} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{A}$$

Where:

- $\mathcal{E}$= induced emf
- $\vec{B}$= magnetic field vector
- $d\vec{A}$= area vector (normal to surface)
- $\int_S \vec{B} \cdot d\vec{A}$= magnetic flux

Magnetic Flux (Φ) in Vector Form

$$\Phi = \int_S \vec{B} \cdot d\vec{A}$$

Flux changes when:

- Magnetic field changes

- Area of the loop changes
- Orientation (angle) changes

2. Differential Form of Faraday's Law

Maxwell upgraded Faraday's law into a differential form:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

This equation means:

- A **changing magnetic field** creates a **circulating electric field**.This is the foundation of **electromagnetic waves**.

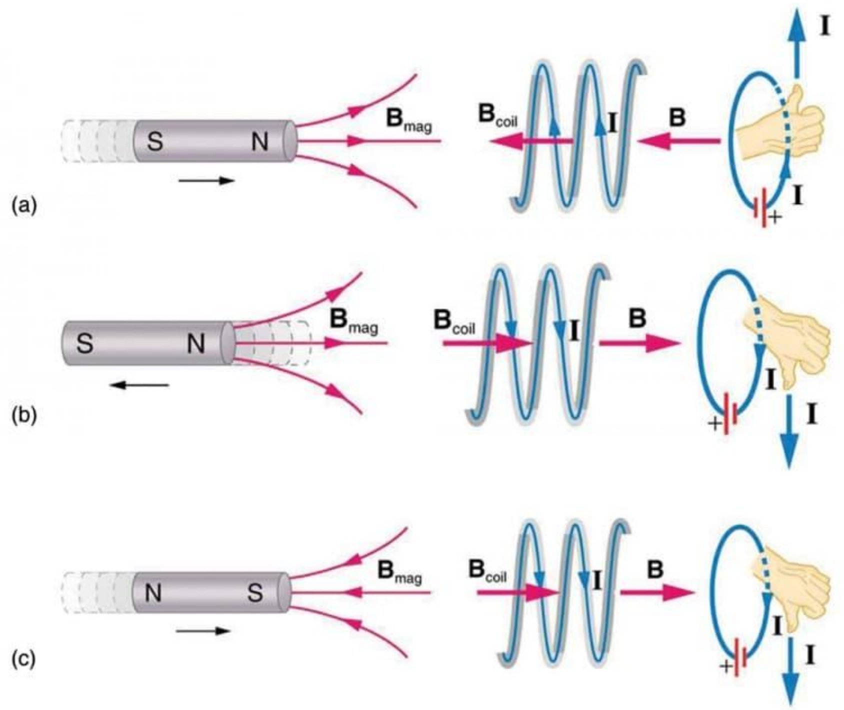


Figure 3. 5

3. Lenz's Law (Direction of Induced EMF)

Lenz's Law states:

The direction of the induced emf is such that it opposes the change in magnetic flux that produced it.

Vector Form (Sign Convention)

Lenz's law is represented by the **negative sign** in Faraday's law:

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

The minus sign indicates **opposition**.

Physical Meaning

- If flux **increases**, the induced current produces a magnetic field that **opposes** increase.
- If flux **decreases**, the induced current produces a magnetic field that **tries to increase it back**

4. Combined Faraday–Lenz Interpretation

When flux through a coil changes:

1. **Magnitude** of emf is given by Faraday
2. **Direction** of emf/current is given by Lenz

Together:

- They conserve energy
- Prevent violation of Newton's laws
- Make electromagnetic devices behave predictably

5. Real-Life Applications

Device	Role of Faraday + Lenz
Generators	Changing flux → emf generation
Transformers	Alternating flux → induced emf in secondary
Induction Stove	Changing magnetic field → eddy current heating
Electric Brakes	Lenz's opposition → braking torque
Metal detectors	Induced eddy currents indicate metal presence

Coefficient of self inductance of solenoid

When current flows through a solenoid, it creates a magnetic field.

If the current changes, the magnetic flux linked with the solenoid also changes.

Because of this change in flux, the solenoid **induces an emf in itself** — this phenomenon is called **self-induction**.

The constant that measures how *strongly* a solenoid opposes a change in current is called its:

Self-Inductance (L)

It basically tells you:

"How much flux does the solenoid produce per unit current?"

Definition of Self-Inductance

A solenoid has self-inductance Lif:

$$L = \frac{\Phi_B}{I}$$

where

Φ_B= magnetic flux linked with the coil

I= current

For an N-turn solenoid:

$$L = \frac{N\Phi_B}{I}$$

Magnetic Field Inside a Solenoid

For a long solenoid with:

- number of turns per unit length $n = \frac{N}{l}$
- current I

the magnetic field inside is:

$$B = \mu_0 \mu_r n I$$

where

μ_0= permeability of free space

μ_r= relative permeability of core material

Magnetic Flux through the Solenoid

Flux through each turn:

$$\Phi = BA = \mu_0 \mu_r n I A$$

Total flux linkage:

$$N\Phi = N(\mu_0 \mu_r n I A)$$

Substitute $n = \frac{N}{l}$:

$$N\Phi = \mu_0 \mu_r \frac{N^2 A I}{l}$$

Self-Inductance Formula

Using:

$$L = \frac{N\Phi}{I}$$

we get:

Final Formula

$$\boxed{L = \mu_0 \mu_r \frac{N^2 A}{l}}$$

where:

- L= self-inductance
- N= number of turns
- A= cross-sectional area
- l= length of solenoid
- $\mu_0 = 4\pi \times 10^{-7}$H/m
- μ_r= relative permeability of core

Interpretation (Super Simple)

- More turns (↑N) → **more inductance**
- Larger area (↑A) → **stronger field → more inductance**
- Longer solenoid (↑l) → **weaker field → less inductance**
- Magnetic core (↑μr) → **massive boost in inductance**

Basically:

Solenoids with many turns, big area, and magnetic cores (iron) have high self-inductance.

SI Unit

$$[L] = \text{Henry (H)}$$

1 Henry = one volt induced when current changes at 1 A per second.

Anderson's Method

Introduction

Anderson's Bridge is an AC bridge used to **measure the self-inductance (L)** of a coil **very accurately**.

It is basically a modified version of **Maxwell's inductance–capacitance bridge**, but with an extra resistor inserted so that the measurement becomes more precise, especially when the inductance is small.

This method is widely used in:

- AC laboratories
- Standard inductance calibration
- Precision measurement of inductors in research setups

The highlight is that it gives **high accuracy** even when the capacitor values are small and resistors are imperfect.

Principle of Anderson's Bridge

The method works on the principle of:

AC bridge balance

At balance:

No current flows through the detector (headphone/galvanometer).

The condition of balance gives an equation that includes the **unknown inductance** $L$of the coil.

By solving that equation, we compute L with high accuracy.

3. Circuit of Anderson's Bridge

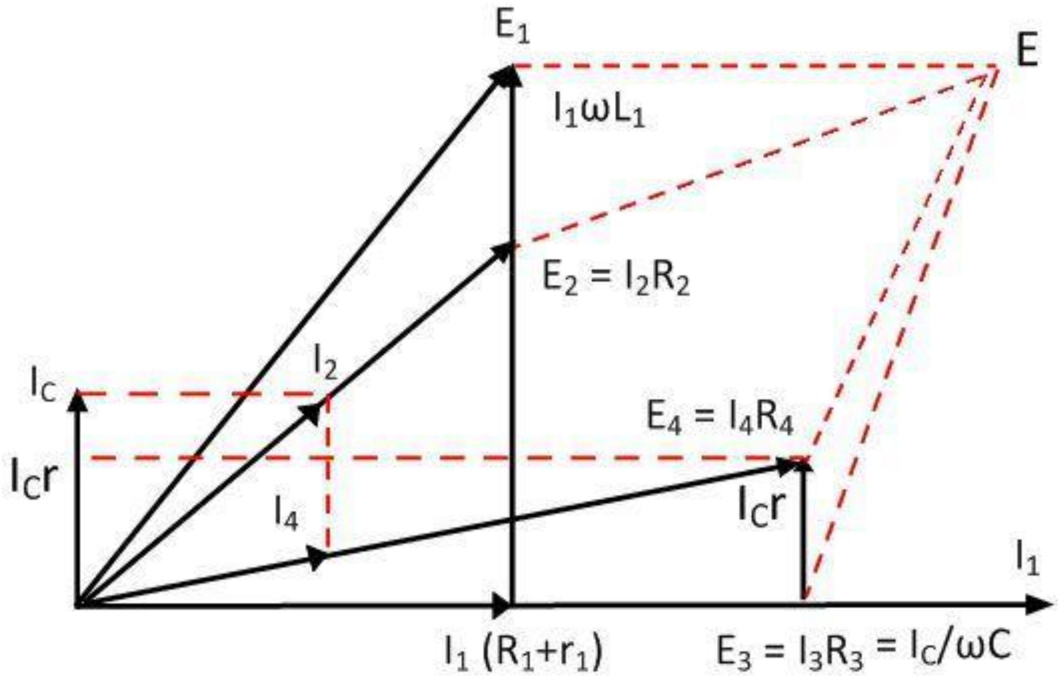


Figure 3. 6

The circuit consists of:

- A standard capacitor C
- Known non-inductive resistors R_1, R_2, R_3, R_4
- A variable resistor r
- An unknown inductance coil with internal resistance R
- A headset or null detector
- An AC source

The unique feature is the **additional resistor r in series with the capacitor**, which makes it more accurate than Maxwell's bridge.

Balance Condition

At the balance point:

- No current flows through the detector
- Voltage drops on different arms follow the AC bridge balance rules

- Complex impedances (due to L and C) equate in opposite arms

The final balance equation for self-inductance is:

$$\boxed{L = C\,(R_1R_4 + R_4r + R_1r)}$$

And the resistance of the inductive coil is:

$$\boxed{R = \frac{R_2}{R_3}R_4}$$

These equations help determine:

- **L → the unknown inductance**
- **R → the coil's resistance**

Maxwell Bridge	Anderson Bridge
Accurate only for larger inductances	Accurate even for small inductances
Requires precise resistors	Uses a capacitor for accuracy
More error-prone	Less error due to extra balancing arm

Advantages of Anderson's Bridge:

- High precision
- Eliminates stray capacitance errors
- Suitable for both low and medium inductances
- Uses standard capacitor values (easy to obtain)

Procedure

1. **Connect the circuit** exactly as per the Anderson bridge diagram.
2. **Set the AC supply** to appropriate frequency (usually 1 kHz).
3. Adjust resistances $R_1, R_2, R_3, R_4$to some initial values.

4. Vary the resistor r until the **detector shows a null** (no sound / no deflection).
5. Note all resistor values and capacitor value C.
6. Substitute in the formula:

$$L = C\,(R_1R_4 + R_4r + R_1r)$$

7. Calculate the self-inductance.

Applications

- Measurement of self-inductance of coils
- Calibration of standard inductors
- Precision AC measurements
- Used in electronics and physics labs
- High accuracy for research in electromagnetism

Coefficient of Mutual Inductance between two Coaxial Solenoids

Introduction

When **two solenoids share the same axis** (coaxial arrangement), the magnetic flux produced by one solenoid passes through the other.

So, if the current in the primary solenoid changes, an emf is induced in the secondary solenoid.

This electromagnetic "interaction" between the two coils is measured by:

Mutual Inductance (M)

It tells us how effectively flux from one solenoid links with the other.

Definition

Mutual inductance Mbetween two coils is defined as:

$$M = \frac{N_2 \Phi_{21}}{I_1}$$

Where:

- I_1= current in primary solenoid
- Φ_{21}= flux through each turn of secondary due to primary
- N_2= number of turns in secondary

This formula basically measures:

"Flux created in secondary for unit current in primary."

Magnetic Field Produced by Primary Solenoid

Consider two long, coaxial solenoids:

- Solenoid 1:
  - N_1turns
  - Length l
  - Cross-sectional area A
  - Turns per unit length $n_1 = \frac{N_1}{l}$

When current I_1flows through solenoid 1:

$$B_1 = \mu_0 \mu_r n_1 I_1$$

Flux Linking the Secondary Solenoid

Since the solenoids are coaxial and have the same area A:

Flux through each turn of secondary coil:

$$\Phi_{21} = B_1 A = \mu_0 \mu_r n_1 I_1 A$$

Total flux linkage of secondary:

$$N_2 \Phi_{21} = N_2(\mu_0 \mu_r n_1 I_1 A)$$

Formula for Mutual Inductance

Using definition:

$$M = \frac{N_2 \Phi_{21}}{I_1}$$

Substitute the flux:

$$M = \mu_0 \mu_r n_1 N_2 A$$

And since $n_1 = \frac{N_1}{l}$:

$$\boxed{M = \mu_0 \mu_r \frac{N_1 N_2 A}{l}}$$

Where:

- N_1, N_2= number of turns
- A= common cross-sectional area
- l= length of solenoid
- μ_0= permeability of free space
- μ_r= relative permeability of core

Physical Meaning

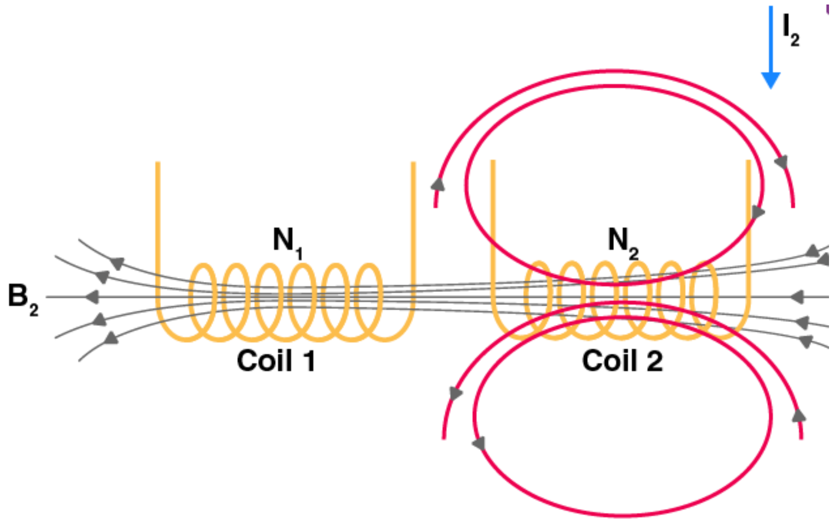


Figure 3. 7

- If the coils share **more area**, M increases.
- If the coils are **close and perfectly coaxial**, flux linkage is maximum.
- If you insert an **iron core**, μ_r increases → M increases significantly.

So mutual inductance can be controlled by:

- Increasing turns
- Increasing area
- Decreasing length
- Using magnetic cores

Special Case: Ideal Coupling

For perfectly linked coaxial solenoids:

$$M = \sqrt{L_1 L_2}$$

This is the highest mutual inductance possible between the two coils.

SI Unit

$$[M] = \text{Henry (H)}$$

Applications

- Transformers
- Inductive charging
- Wireless power transfer
- Induction motors
- Communication circuits

Coefficient of coupling

Coefficient of coupling (k) — definition and meaning

The **coefficient of coupling** k between two coils measures how effectively the magnetic flux produced by one coil links (passes through) the other coil.

Two equivalent common definitions:

1. In terms of inductances and mutual inductance:

$$\boxed{k = \frac{M}{\sqrt{L_1 L_2}}}$$

where M is the mutual inductance, L_1 and L_2 are the self-inductances of coil 1 and coil 2 respectively.

2. In terms of flux linkages:

$$\boxed{k = \frac{\Phi_{12}}{\sqrt{\Phi_{11}\,\Phi_{22}}} = \frac{\Phi_{21}}{\sqrt{\Phi_{11}\,\Phi_{22}}}}$$

where

- $\Phi_{11}$is the flux through one turn of coil 1 due to current I_1(self flux per turn of coil 1),
- $\Phi_{22}$is the flux through one turn of coil 2 due to current I_2(self flux per turn of coil 2),
- $\Phi_{12}$is flux through one turn of coil 2 due to current I_1, and $\Phi_{21}$is flux through one turn of coil 1 due to I_2. For linear media $\Phi_{12} = \Phi_{21}$per unit current symmetry holds.

Range: $0 \leq k \leq 1$.

- $k = 1$→ perfect coupling (all flux of one links the other).
- $k = 0$→ no coupling (no shared flux).

Derivation from flux linkages → $M = k\sqrt{L_1 L_2}$

Start with definitions:

Self inductances:

$$L_1 = \frac{N_1 \Phi_{11}}{I_1}, L_2 = \frac{N_2 \Phi_{22}}{I_2}.$$

Mutual inductance (flux in coil 2 due to current in coil 1):

$$M = \frac{N_2 \Phi_{12}}{I_1} = \frac{N_1 \Phi_{21}}{I_2}.$$

Define coupling by flux ratio:

$$k = \frac{\Phi_{12}}{\sqrt{\Phi_{11}\Phi_{22}}}.$$

Multiply both sides of that definition by N_2/I_1:

$$\frac{N_2 \Phi_{12}}{I_1} = k\,\frac{N_2\sqrt{\Phi_{11}\Phi_{22}}}{I_1}.$$

But left hand side is M. Replace $\Phi_{11} = L_1 I_1 / N_1$and $\Phi_{22} = L_2 I_2 / N_2$(or use symmetric manipulation). After algebra you obtain

$$\boxed{M = k\sqrt{L_1 L_2}}.$$

This is the standard relation used in circuit theory.

Energy argument and inequality $|M| \leq \sqrt{L_1 L_2}$

Magnetic energy stored in two coupled coils (linear, passive) is

$$W = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2.$$

For any real currents I_1, I_2, the stored energy must be non-negative. Consider currents chosen so that one can force the quadratic form to show the discriminant condition. Requiring positive definiteness leads to

$$L_1 L_2 - M^2 \geq 0 \Rightarrow |M| \leq \sqrt{L_1 L_2}.$$

Define $k$by dividing both sides by $\sqrt{L_1 L_2}$, giving $|k| \leq 1$. For passive reciprocal coils $k \geq 0$.

This explains mathematically why $k$is bounded by 1.

Physical interpretation

- Mmeasures actual flux linkage between coils for a unit current.
- $\sqrt{L_1 L_2}$is the geometric mean of each coil's ability to produce flux (self coupling).
- $k$is the fraction of the "possible" linkage that actually occurs.

If part of the flux produced by coil 1 leaks (doesn't link coil 2), coupling is reduced.

Coaxial solenoids special case (simple formula and example)

Consider two long coaxial solenoids with same length l, common area A, turns N_1, N_2 and same core permeability μ. For ideal long solenoids:

Self inductances:

$$L_1 = \mu \frac{N_1^2 A}{l}, L_2 = \mu \frac{N_2^2 A}{l}.$$

Mutual inductance (all flux of one links the other) :

$$M = \mu \frac{N_1 N_2 A}{l}.$$

Then

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{\mu \frac{N_1 N_2 A}{l}}{\sqrt{\mu \frac{N_1^2 A}{l} \mu \frac{N_2^2 A}{l}}} = \frac{\mu N_1 N_2 A/l}{\mu N_1 N_2 A/l} = 1.$$

So ideal coaxial solenoids (perfectly overlapping, same core, no leakage) give $k = 1$. In practice geometry, finite length and leakage reduce k slightly below 1.

Relation to leakage inductances (practical circuit model)

In transformer or coupled-coil modelling it is common to split each coil's flux into linked (mutual) and leakage parts:

$$L_1 = L_{m1} + L_{\ell 1}, L_2 = L_{m2} + L_{\ell 2},$$

where L_{m1}, L_{m2} are the parts that contribute to mutual flux (ideally $L_{m1} = L_{m2} = M$ per turn convention), and L_ℓ are leakage inductances whose flux does not link the other coil. Then

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{L_m}{\sqrt{(L_m + L_{\ell 1})(L_m + L_{\ell 2})}}.$$

This shows leakage inductance reduces k.

How to measure k experimentally

Typical laboratory methods:

1. **Measure L_1 and L_2** individually (with open secondary or using L-meter).
2. **Measure mutual inductance** M (by measuring induced emf in secondary for known dI_1/dt, or using bridge methods).
3. Compute $k = M/\sqrt{L_1 L_2}$.

Alternative practical tests on transformers: measure open-circuit and short-circuit impedances to infer leakage and magnetizing inductances, and hence approximate coupling.

Role in circuits and transformers

- High $k(\approx 1) \rightarrow$ efficient energy transfer (transformers, tight inductive coupling).
- Low $k \rightarrow$ weak coupling (loosely coupled coils, wireless charging at distance uses low-to-moderate k tuned with resonance).
- In two-coil circuit equations: coupling appears in mutual impedance $j\omega M$ and affects series/parallel resonance, bandwidth, isolation.

Energy viewpoint (alternate expression for k)

When only coil 1 carries current I_1 and coil 2 open, the flux linking coil 1 is Φ_{11} and part of that flux Φ_{12} links coil 2. Then one can show

$$k^2 = \frac{\Phi_{12}\Phi_{21}}{\Phi_{11}\Phi_{22}} = \frac{M^2}{L_1 L_2},$$

consistent with earlier definitions.

Dependence on geometry and materials

Factors that increase k:

- Greater overlap of coil areas (coaxial alignment).

- Smaller separation between coils.
- Use of high permeability core linking both coils (iron core in a transformer).
- Larger common cross-sectional area.

Factors that reduce k: misalignment, axial separation, shielding, separate cores, or significant portion of flux that leaks.

Typical values and examples

- Perfectly wound, tightly coupled transformer cores: $k \approx$ 0.98–0.999 (very close to 1).
- Air-core loosely coupled coils (distance several times coil radius): $k$can be 0.1–0.5 or lower.
- Resonant wireless power transfer systems often operate with $k$in 0.1–0.5 range and use Q-factor to boost power transfer.

Unit 4: Transisent and Alternating Currents

1. Introduction
2. Growth and decay of current in a circuit containing resistance and inductance
3. Growth and decay of charge in a circuit containing Resistance and Capacitor
4. Growth and decay of charge in an LCR Circuit (Expression for charge only)
5. Peak average and rms value of AC
6. LCR Series
7. Parallel Circuits & Resonance condition
8. Q Factor
9. Power factor

Introduction

In electrical circuits, currents do not always attain a steady value immediately after a voltage is applied. Many practical situations involve currents that change with time. To understand such time-dependent behaviour, the study of **transient currents** and **alternating currents** becomes essential in physics and electrical engineering.

1. Need for Studying Time-Dependent Currents

When a circuit containing resistance (R), inductance (L), or capacitance (C) is suddenly connected to or disconnected from a power supply, the current and voltage do not change instantaneously.

Because inductors and capacitors have the ability to store and release energy, they oppose sudden changes in current or voltage. This gives rise to **transient phenomena**.

Examples:

- Current rising gradually in an RL circuit when the switch is closed
- Charge building up on a capacitor in an RC circuit
- Current oscillating between L and C in an LC circuit

These time-dependent processes are essential to understand the working of many modern electrical systems such as filters, oscillators, power supplies, and communication devices.

2. Transient Currents

A **transient current** is a temporary current that exists for a limited duration of time until the circuit reaches a steady state.

In circuits containing L and C, transients appear due to the storage of energy:

- **Inductors store energy in the magnetic field**, resisting sudden changes in current.
- **Capacitors store energy in the electric field**, resisting sudden changes in voltage.

Transient currents typically decay exponentially with time, governed by time constants such as:

- $\tau = \frac{L}{R}$ for RL circuits
- $\tau = RC$ for RC circuits

Understanding these transients helps in the design of stable and reliable electronic circuits.

3. Alternating Currents (A.C.)

Unlike transient currents, **alternating currents** vary continuously with time, even in steady-state conditions.

An AC changes its magnitude and direction periodically, most commonly in the form of a sine wave.

The standard mathematical form of an alternating current is:

$$i(t) = I_0 \sin(\omega t)$$

where

I_0— maximum or peak current

ω— angular frequency

t— time

Alternating currents are widely used in electrical power systems because:

- They can be transmitted efficiently over long distances
- Voltage levels can be easily stepped up or down using transformers

4. Importance of Transient and AC Analysis

Understanding transient and alternating currents is crucial because:

1. It explains how circuits behave immediately after switching operations.
2. It helps in designing circuits that require precise control of current and voltage waveforms.
3. It is fundamental for understanding resonance, impedance, reactance, power factor, and energy losses in AC circuits.
4. It lays the foundation for advanced topics like signal processing, communication systems, and power electronics.

5. Scope of the Chapter

This chapter will cover:

- Growth and decay of current in RL circuits
- Charging and discharging of capacitors in RC circuits
- Oscillatory behaviour of LC circuits
- Differential equation method for transient analysis
- Nature of sinusoidal AC
- Phase relationships between voltage and current
- Impedance of R, L, and C in AC circuits

- Power in AC circuits and power factor
- Resonance in LCR circuits

Growth and decay of current in a circuit containig resistance and inductance

When an electric circuit contains a **resistor (R)** and an **inductor (L)** connected in series with a source of EMF, the current does **not** reach its maximum value instantaneously.

This delay occurs because:

- A **resistor** opposes the flow of current.
- An **inductor** opposes any *change* in current (due to its back EMF).

Therefore, when the switch is closed, the current **grows gradually**.

When the battery is removed, the current **decays gradually**.

These time-varying processes are called **transients**, and the analysis is essential for understanding practical circuits like motors, relays, transformers, and switching devices.

Circuit Description

- A resistor $R$and an inductor $L$are connected in series with a battery of EMF E.
- At the instant the switch is closed, the current starts from **zero** and rises gradually towards its maximum value.

Reason for Gradual Growth

The inductor produces a **back EMF** opposing the change in current:

$$e_L = -L\frac{di}{dt}$$

This back EMF initially cancels the applied EMF. As time increases, $\frac{di}{dt}$decreases, so the back EMF falls, allowing current to rise.

Derivation of Current Growth Equation

At any instant:

$$E = iR + L\frac{di}{dt}$$

Rearranging:

$$L\frac{di}{dt} = E - iR$$

$$\frac{di}{E - iR} = \frac{R}{L}dt$$

Integrating:

$$\int_0^i \frac{di}{E - iR} = \frac{R}{L}\int_0^t dt$$

$$-\frac{1}{R}\ln(E - iR) = \frac{R}{L}t + C$$

Applying initial condition (at $t = 0$, $i = 0$):

$$C = -\ln(E)$$

After simplification:

$$i = \frac{E}{R}(1 - e^{-\frac{R}{L}t})$$

Final Expression for Growth of Current

$$\boxed{i(t) = I_0(1 - e^{-t/\tau})}$$

where,

- $I_0 = \frac{E}{R} \longrightarrow$ final steady current

- $\tau = \frac{L}{R} \rightarrow$ **time constant**

Time Constant (τ)

$$\boxed{\tau = \frac{L}{R}}$$

Meaning:

- It is the time taken for current to reach **63%** of its maximum value.
- Larger Lor smaller $R \rightarrow$ slower growth.

2. Decay of Current in an R–L Circuit

Circuit Description

When the battery is disconnected:

- The inductor tries to **maintain the current** by releasing stored magnetic energy.
- Current gradually decays to zero.

Derivation of Decay Equation

During decay:

$$L\frac{di}{dt} + iR = 0$$

$$\frac{di}{i} = -\frac{R}{L}dt$$

Integrating:

$$\ln i = -\frac{R}{L}t + C$$

When $t = 0$, $i = I_0$:

$$C = \ln I_0$$

Thus:

$$\boxed{i(t) = I_0 e^{-t/\tau}}$$

This shows an **exponential decrease** of current with time.

Graph of Growth and Decay

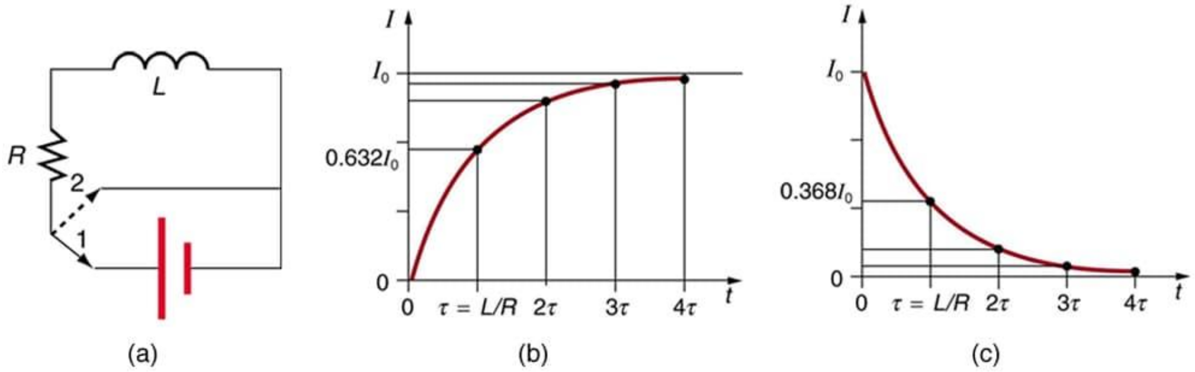


Figure 4. 1

Key Features

- Growth:

$$i(t) = I_0(1 - e^{-t/\tau})$$

Decay:

$$i(t) = I_0 e^{-t/\tau}$$

- The curve approaches I_0or 0 **asymptotically**.
- Time constant τ gives the speed of response.

3. Physical Interpretation

Growth Phase

- Magnetic field builds up in the inductor.
- Energy stored increases:

$$U = \frac{1}{2} L i^2$$

Decay Phase

- The inductor releases energy into the resistor.
- Current falls and magnetic energy dissipates as heat.

Applications

- Electric motor start-up
- Relay and solenoid circuits
- SMPS and Power electronics
- Communication circuits
- Filter and choke coils
- Automotive ignition systems

Growth and decay of charge in a circuit containing Resistance and Capacitor

A capacitor is an electrical component that **stores energy in the form of electric charge**.

When connected in series with a resistor and a voltage source, the capacitor does **not** charge or discharge instantly.

This delay occurs because:

- The **resistor controls the rate** at which charge flows.
- The **capacitor opposes sudden changes in voltage**, just as an inductor opposes sudden changes in current.

Therefore, the amount of charge on the capacitor and the current in the circuit vary **exponentially** with time.

These time-dependent processes are called **transients**, and understanding them is essential for analysing filters, timing circuits, sensors, digital electronics, and communication systems.

Circuit Description

- A resistor $R$and capacitor $C$are connected in series with a battery of EMF E.
- Initially, the capacitor is uncharged.
- When the switch is closed, charge begins to accumulate on the capacitor plates.

Reason for Gradual Charging

As charge builds up, the voltage across the capacitor:

$$V_C = \frac{q}{C}$$

opposes the battery voltage.

Thus, the charging current gradually decreases.

Derivation of Charge Growth Equation

At any instant, using Kirchhoff's law:

$$E = iR + \frac{q}{C}$$

Since $i = \frac{dq}{dt}$:

$$E = R\frac{dq}{dt} + \frac{q}{C}$$

Rearranging:

$$R\frac{dq}{dt} = E - \frac{q}{C}$$

$$\frac{dq}{E - \frac{q}{C}} = \frac{dt}{RC}$$

Integrating:

$$\int_0^q \frac{dq}{E - \frac{q}{C}} = \frac{1}{RC}\int_0^t dt$$

Solving gives:

$$q = CE(1 - e^{-t/RC})$$

Final Expression for Growth of Charge (Charging)

$$\boxed{q(t) = Q_0(1 - e^{-t/\tau})}$$

where,

- $Q_0 = CE \rightarrow$ maximum (steady-state) charge
- $\tau = RC \rightarrow$ **time constant**

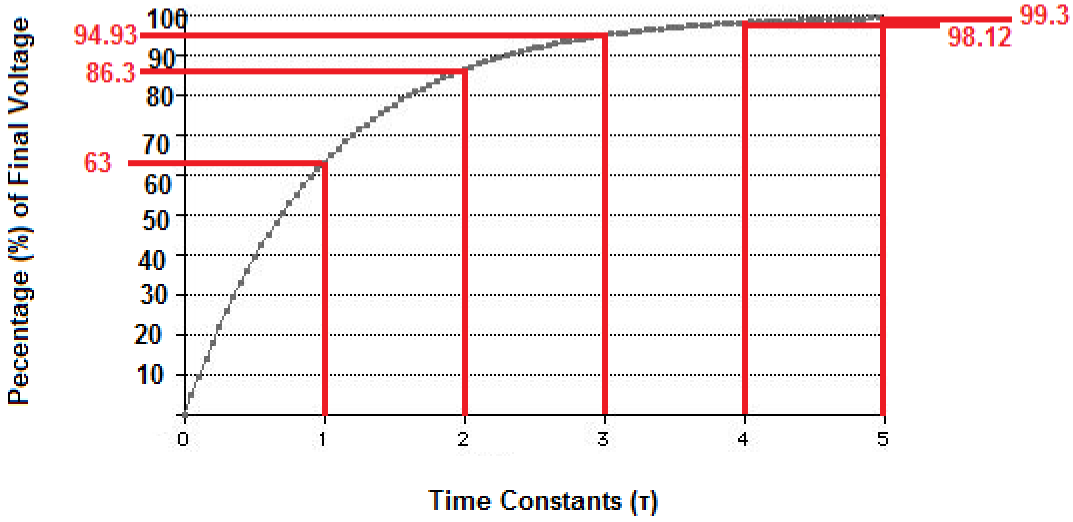


Figure 4. 2

Time Constant (τ = RC)

Meaning of τ:

- The time taken for the capacitor charge to reach **63%** of full charge.
- A larger R or C makes charging **slower**.
- A smaller R or C makes charging **faster**.

2. Decay of Charge in an R–C Circuit (Discharging)

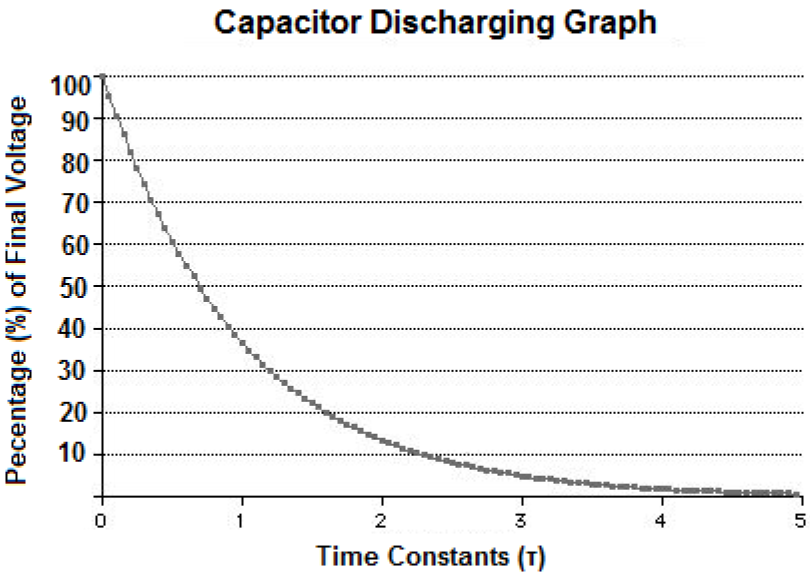


Figure 4. 3

Circuit Description

Once the battery is removed:

- The capacitor releases its stored charge through the resistor.
- The charge and voltage drop exponentially.
- Current flows only until the capacitor becomes fully discharged.

Derivation of Charge Decay Equation

During discharge, the loop equation is:

$$R\frac{dq}{dt} + \frac{q}{C} = 0$$

$$\frac{dq}{q} = -\frac{dt}{RC}$$

Integrating:

$$\ln q = -\frac{t}{RC} + \ln Q_0$$

Thus the charge at any time t:

$$\boxed{q(t) = Q_0 e^{-t/\tau}}$$

This shows **exponential decay** of charge.

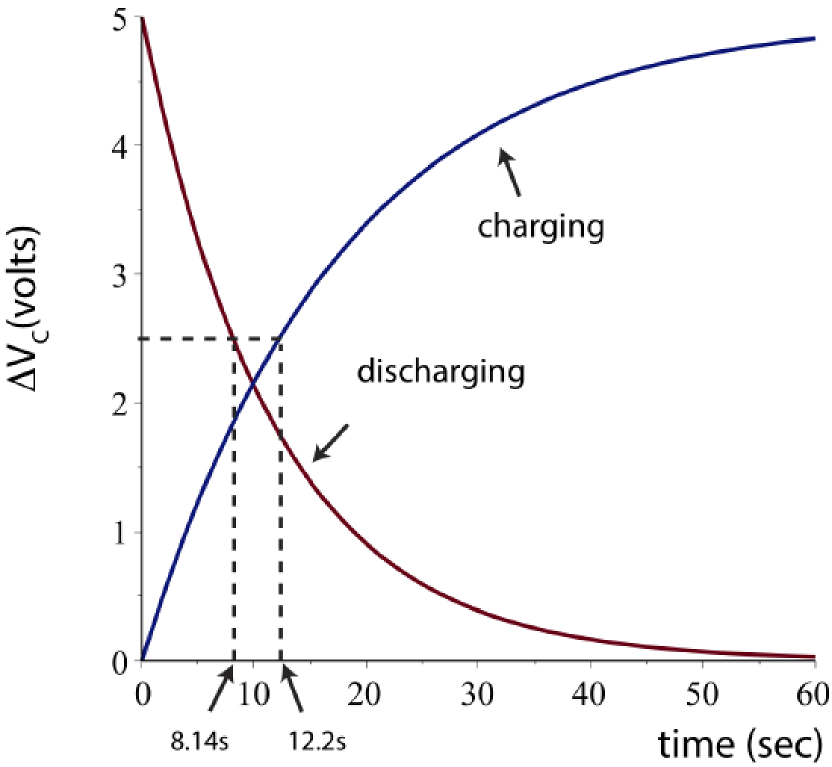


Figure 4. 4

3. Graph of Capacitor Charging and Discharging

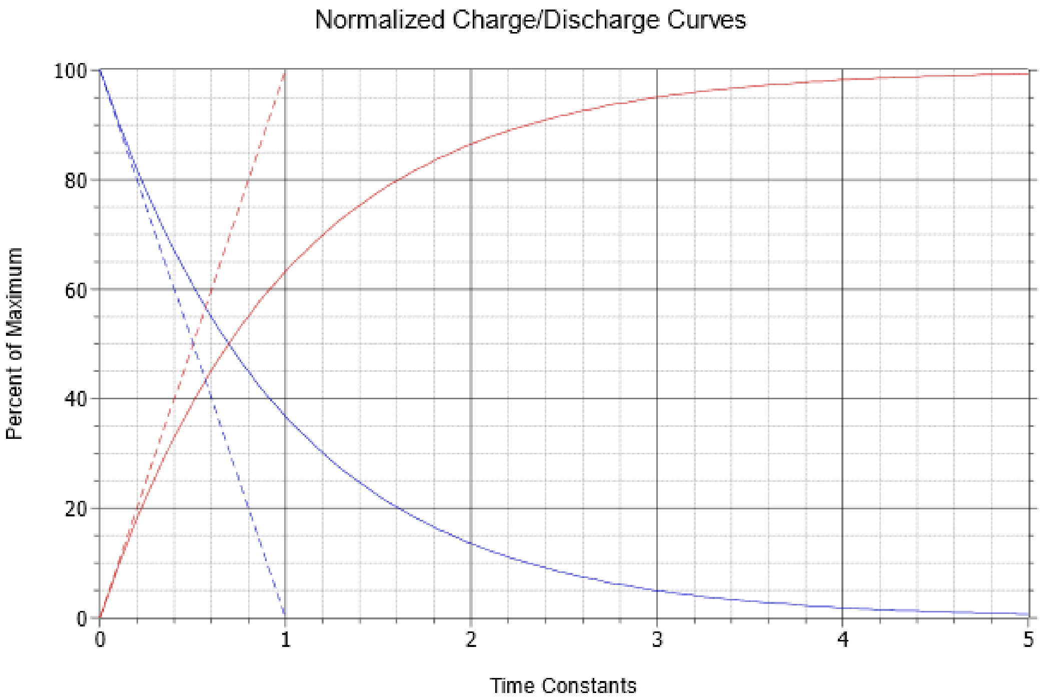


Figure 4. 5

Charging curve

$$q(t) = Q_0(1 - e^{-t/\tau})$$

Discharging curve

$$q(t) = Q_0 e^{-t/\tau}$$

These curves show:

- Slow start
- Rapid change around τ
- Asymptotic approach to final values

Physical Significance

Charging

- Electric energy is stored in the capacitor:

$$U = \frac{1}{2}CV^2$$

- Current is maximum at $t = 0$and decreases to zero as capacitor fills.

Discharging

- Stored energy flows through the resistor and is dissipated as heat.
- Current is maximum at $t = 0$and decays exponentially.

Applications of R–C Transients

- Filters in communication circuits
- Time delay circuits
- Camera flash circuits
- Integrators and differentiators
- Pulse forming networks

- Heart pacemakers
- Sensors and timing modules (555 timer)

Growth and decay of charge in an LCR Circuit (Expression for charge only)

Consider a series circuit containing an inductor L, a capacitor $C$and a resistor R. For the *natural* (free) response we assume the external source is removed (or zero) and the circuit is left to evolve from initial capacitor charge $q(0) = Q_0$and initial current $i(0) = \dot{q}(0) = I_0$. Energy initially stored in $L$and/or Ccauses the circuit to respond and the resistor dissipates energy, producing decay of oscillations or exponential decay depending on damping. See typical damped responses.

Governing differential equation

By applying Kirchhoff's voltage law around the loop (sign convention consistent with $i = \dot{q}$):

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0.$$

Divide by $L$and define standard parameters:

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}\,q = 0.$$

Introduce the damping coefficient $\alpha$and natural (undamped) angular frequency ω_0:

$$\boxed{2\alpha = \frac{R}{L} \Rightarrow \alpha = \frac{R}{2L}\ , \omega_0 = \frac{1}{\sqrt{LC}}}$$

So the standard form is

$$\boxed{\ddot{q} + 2\alpha\dot{q} + \omega_0^2 q = 0}.$$

This is a linear, constant-coefficient second-order homogeneous ODE.

Characteristic equation and modes

Characteristic equation:

$$s^2 + 2\alpha s + \omega_0^2 = 0 \Rightarrow s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}.$$

Let $\Delta = \alpha^2 - \omega_0^2$. The sign of Δgives three cases:

- **Underdamped:** $\alpha < \omega_0 \Rightarrow \Delta < 0 \rightarrow$ complex conjugate roots.
- **Critically damped:** $\alpha = \omega_0 \Rightarrow \Delta = 0 \rightarrow$ repeated real root.
- **Overdamped:** $\alpha > \omega_0 \Rightarrow \Delta > 0 \rightarrow$ two distinct real negative roots.

Representative plots and envelopes are shown in the references.

Solutions (expressions for $q(t)$)

We supply the closed-form $q(t)$in each damping regime for initial conditions $q(0) = Q_0$and $\dot{q}(0) = I_0$.

A — Underdamped case ($\alpha < \omega_0$)

Define the *damped angular frequency*

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}.$$

General solution (real form):

$$q(t) = e^{-\alpha t}[A\cos(\omega_d t) + B\sin(\omega_d t)].$$

Apply initial conditions $q(0) = Q_0$and $\dot{q}(0) = I_0$:

- $A = Q_0$.
- From derivative at $t = 0$:

$$\dot{q}(0) = -\alpha A + B\omega_d = I_0 \Rightarrow B = \frac{I_0 + \alpha Q_0}{\omega_d}.$$

So the explicit expression is

$$\boxed{q(t) = e^{-\alpha t}[Q_0\cos(\omega_d t) + \frac{I_0 + \alpha Q_0}{\omega_d}\sin(\omega_d t)], (\alpha < \omega_0).}$$

This is an exponentially decaying sinusoid (ringing).

B — Critically damped case ($\alpha = \omega_0$)

Characteristic root $s = -\alpha$(double). General solution:

$$q(t) = (A + Bt)\, e^{-\alpha t}.$$

Initial conditions give:

- $A = Q_0$.
- $\dot{q}(0) = B - \alpha A = I_0 \Rightarrow B = I_0 + \alpha Q_0$.

Hence

$$\boxed{q(t) = [\, Q_0 + (I_0 + \alpha Q_0)\, t\,]\, e^{-\alpha t}, (\alpha = \omega_0).}$$

This decays to zero without oscillation and is the fastest non-oscillatory return to equilibrium.

C — Overdamped case ($\alpha > \omega_0$)

Let $r = \sqrt{\alpha^2 - \omega_0^2}$(real, positive). Roots:

$$s_1 = -\alpha + r, s_2 = -\alpha - r\text{(both negative)}.$$

General solution:

$$q(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}.$$

Use initial conditions $q(0) = Q_0$ and $\dot{q}(0) = I_0$ to solve for constants:

$$\begin{cases} C_1 + C_2 = Q_0, \\ C_1 s_1 + C_2 s_2 = I_0. \end{cases}$$

Solving,

$$\boxed{C_1 = \frac{I_0 - s_2 Q_0}{s_1 - s_2}, C_2 = \frac{s_1 Q_0 - I_0}{s_1 - s_2}}$$

so

$$\boxed{q(t) = \frac{I_0 - s_2 Q_0}{s_1 - s_2} e^{s_1 t} + \frac{s_1 Q_0 - I_0}{s_1 - s_2} e^{s_2 t}, (\alpha > \omega_0).}$$

No oscillation occurs; the response is the sum of two decaying exponentials. (One exponent usually decays faster than the other.)

Remarks

- The resistor causes energy dissipation; the exponential factors (or real negative roots) ensure the charge decays to zero as $t \to \infty$ for the homogeneous case (no sustained source).
- The characteristic time scale for envelope decay is $1/\alpha = 2L/R$ (roughly the e-folding time for amplitude). For underdamped oscillations the envelope is $e^{-\alpha t}$ and oscillations occur at ω_d.
- If a forcing source is present (forced response), the complete solution = homogeneous solution above (transient) + particular steady-state (forced) solution; after long time only the steady-state remains for a driven sinusoid.

- Typical plots of underdamped/critically damped/overdamped responses are shown above for intuition.

Peak average and rms value of AC

An alternating current (AC) is an electric current that **changes its magnitude and direction periodically**.

The most common form of AC is the **sinusoidal waveform**, represented as:

$$i(t) = I_0 \sin \omega t$$

where

- I_0= peak (maximum) value of current
- ω= angular frequency
- t= time

Since AC varies with time, we need special measures to express its **effective value**. The commonly used measures are:

1. **Peak value (Maximum value)**
2. **Average value**
3. **Root Mean Square (RMS) value**

Peak Value (Maximum Value)

Definition

The **peak value** of AC is the **maximum instantaneous value** attained by the alternating current or voltage during one cycle.

For the current:

$$i(t) = I_0 \sin \omega t$$

Peak value is:

$$\boxed{I_0}$$

Similarly, for voltage:

$$v(t) = V_0 \sin \omega t \Rightarrow \boxed{V_0}$$

Importance

- Determines the maximum stress on insulation
- Determines breakdown limits
- Crucial in designing electronic components

Typical sinusoid with peak amplitude is shown below.

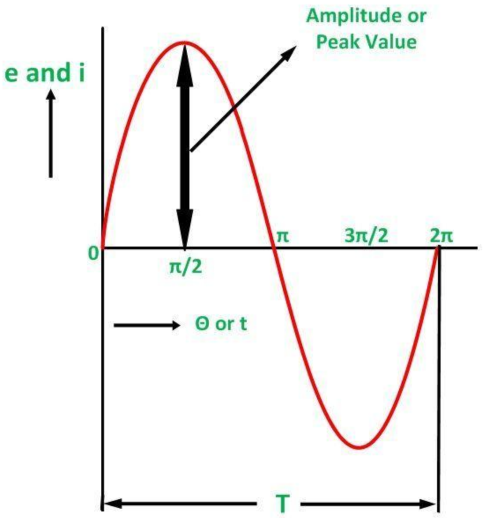


Figure 4. 6

Average Value of AC

Definition

The average value of an AC over one complete cycle is **zero**, because positive and negative halves cancel out.

Therefore, average value is taken over **one half cycle**.

For sinusoidal AC:

$$i(t) = I_0 \sin \omega t$$

Average value over half cycle:

$$I_{\text{avg}} = \frac{1}{\pi} \int_0^{\pi} I_0 \sin \theta \, d\theta$$

$$I_{\text{avg}} = \frac{I_0}{\pi}[-\cos\theta]_0^{\pi}$$

$$I_{\text{avg}} = \frac{I_0}{\pi}(-\cos\pi + \cos 0)$$

$$I_{\text{avg}} = \frac{I_0}{\pi}(1 + 1)$$

$$\boxed{I_{\text{avg}} = \frac{2I_0}{\pi}}$$

Similarly for voltage:

$$\boxed{V_{\text{avg}} = \frac{2V_0}{\pi}}$$

Numerical Ratio

$$\frac{2}{\pi} = 0.637$$

Thus,

$$I_{\text{avg}} = 0.637\, I_0$$

RMS Value of AC (Effective Value)

Definition

The RMS (Root Mean Square) value of AC is the value of the **steady DC current** that would produce the **same amount of heat** in a resistor as the AC produces over one complete cycle.

For a sinusoidal current:

$$i(t) = I_0 \sin\omega t$$

RMS value:

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

Substituting:

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T I_0^2 \sin^2 \omega t\,dt}$$

$$I_{\text{rms}} = I_0\sqrt{\frac{1}{T}\int_0^T \frac{1-\cos 2\omega t}{2}\,dt}$$

$$I_{\text{rms}} = I_0\sqrt{\frac{1}{2}}$$

$$\boxed{I_{\text{rms}} = \frac{I_0}{\sqrt{2}}}$$

Similarly for voltage:

$$\boxed{V_{\text{rms}} = \frac{V_0}{\sqrt{2}}}$$

Numerical Ratio

$$\frac{1}{\sqrt{2}} = 0.707$$

Thus,

$$I_{\text{rms}} = 0.707\,I_0$$

AC RMS representation is shown below.

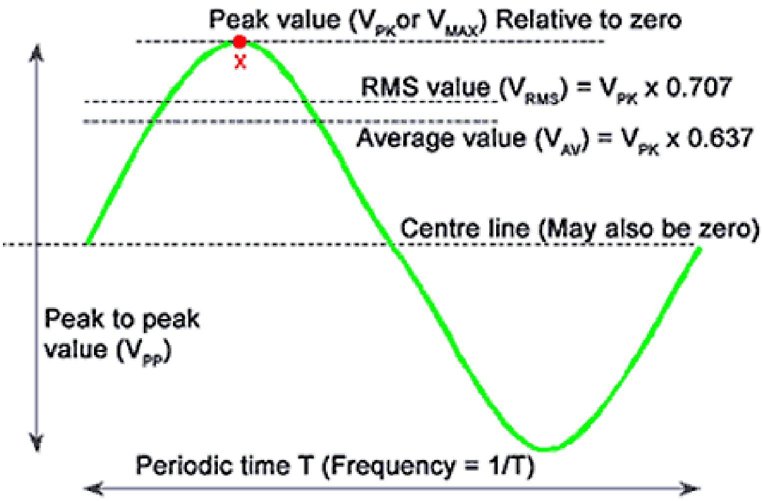


Figure 4. 7

Quantity	Formula (Current)	Numerical Value	Meaning
Peak	I_0	—	Maximum current
Average	$I_{\text{avg}} = \frac{2I_0}{\pi}$	$0.637I_0$	Average over half cycle
RMS	$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$	$0.707I_0$	Effective heating value

Diagram Comparing Peak, Average and RMS

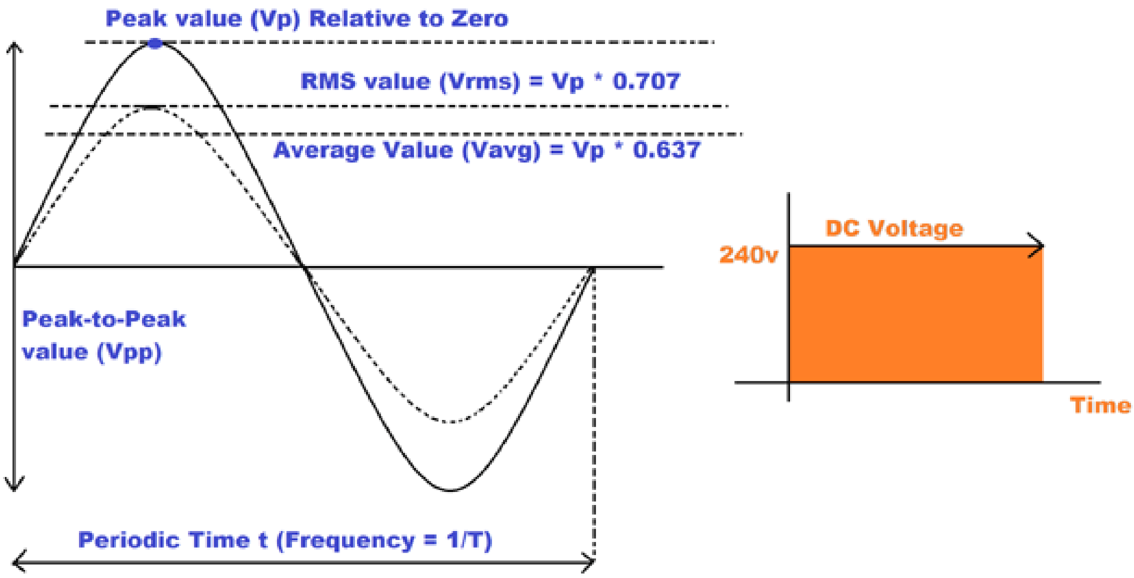


Figure 4. 8

LCR Series

A **series L–C–R circuit** consists of an inductor L, capacitor C, and resistor Rconnected in series with an alternating voltage source. This is one of the most important AC circuits used to understand impedance, phase difference, power factor, and resonance.

Because all components are in series:

- The **same current** flows through R, L, and C.
- The **voltages** across each component differ in magnitude and phase.

Voltages Across Individual Elements

1. Resistor (R)

$$v_R = iR$$

Voltage is **in phase** with current.

2. Inductor (L)

$$v_L = L\frac{di}{dt}$$

Inductor voltage **leads current by** 90°.

3. Capacitor (C)

$$v_C = \frac{1}{C}\int i\,dt$$

Capacitor voltage **lags current by** 90°.

Kirchhoff's Voltage Law

$$v(t) = v_R + v_L + v_C$$

Phasor Representation

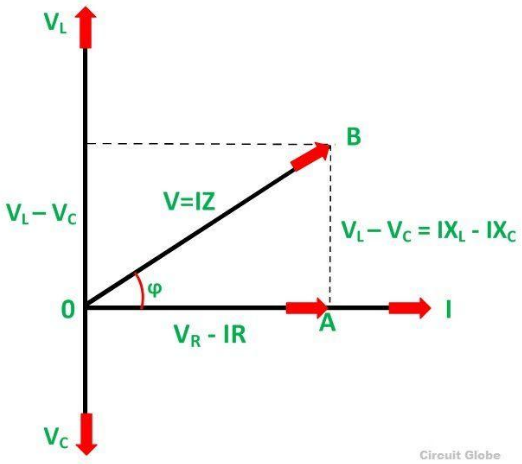


Figure 4. 9

Representing AC voltages as phasors simplifies analysis:

- $V_R$along the current axis
- $V_L$above (leading)
- $V_C$below (lagging)

Net reactive voltage:

$$V_X = V_L - V_C$$

Reactances

$$X_L = \omega L, X_C = \frac{1}{\omega C}$$

- $X_L$increases with frequency
- $X_C$decreases with frequency

Impedance of the Series LCR Circuit

$$Z = R + j(X_L - X_C)$$

Magnitude:

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

Current:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{|Z|}$$

Phase Angle

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

- If $X_L > X_C$: Inductive → voltage leads
- If $X_L < X_C$: Capacitive → voltage lags
- If $X_L = X_C$: Purely resistive

Resonance in Series LCR Circuit

Resonance occurs when:

$$X_L = X_C$$

Thus,

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

At resonance:

- Impedance $Z = R$(minimum)
- Current is **maximum**

- Circuit behaves like a pure resistor
- Voltage and current are in phase

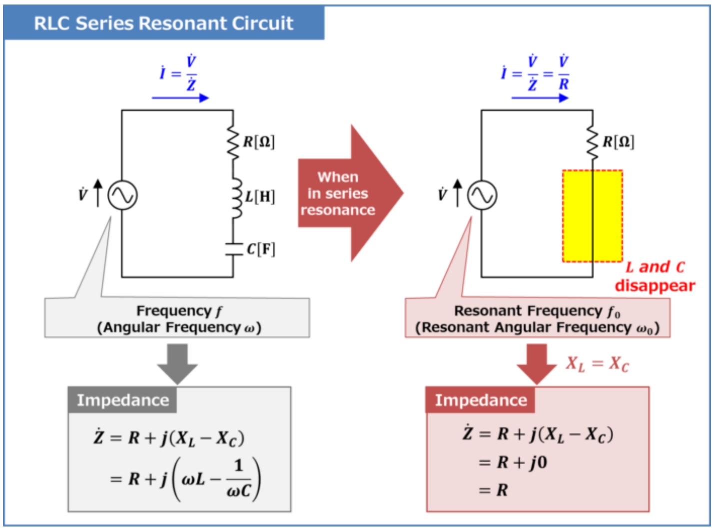


Figure 4. 10

- Voltage across L and C may become very large (voltage magnification)

Quality Factor (Q)

$$Q = \frac{\omega_0 L}{R}$$

High Q:

- Sharp resonance
- Narrow bandwidth
- Used in radio tuning circuits

Bandwidth:

$$\Delta\omega = \omega_2 - \omega_1$$

$$Q = \frac{\omega_0}{\Delta\omega}$$

Power in an LCR Series Circuit

Average power:

$$P = V_{\text{rms}} I_{\text{rms}} \cos\phi = I_{\text{rms}}^2 R$$

- **Only the resistor** consumes real power
- $L$and Cexchange energy with the source but do not dissipate power

Parallel Circuits & Resonance condition

Introduction

A **parallel L–C–R circuit** consists of an inductor L, a capacitor C, and a resistor Rconnected in parallel across an AC supply. Unlike series circuits where **voltage is common**, in a parallel circuit:

- **Voltage across each branch is the same**
- **Currents through the branches differ**

Resonance in a parallel circuit is called **current resonance** because the total line current becomes minimum at a particular frequency.

Branch Currents

Let the supply voltage be:

$$v(t) = V_m \sin\omega t$$

Then the branch currents are:

(a) Current through inductor

$$I_L = \frac{V}{\omega L}$$

Current **lags voltage by** 90°.

(b) Current through capacitor

$$I_C = V\omega C$$

Current **leads voltage by** 90°.

(c) Current through resistor

$$I_R = \frac{V}{R}$$

Current is **in phase** with voltage.

Condition for Resonance in Parallel Circuit

For resonance,

The net reactive current must be zero

$$I_C = I_L$$

Because capacitive current leads by $+90^\circ$and inductive current lags by -90°, they cancel each other when equal.

So,

$$\omega C = \frac{1}{\omega L}$$

This gives the **resonant angular frequency**:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

And resonant frequency:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

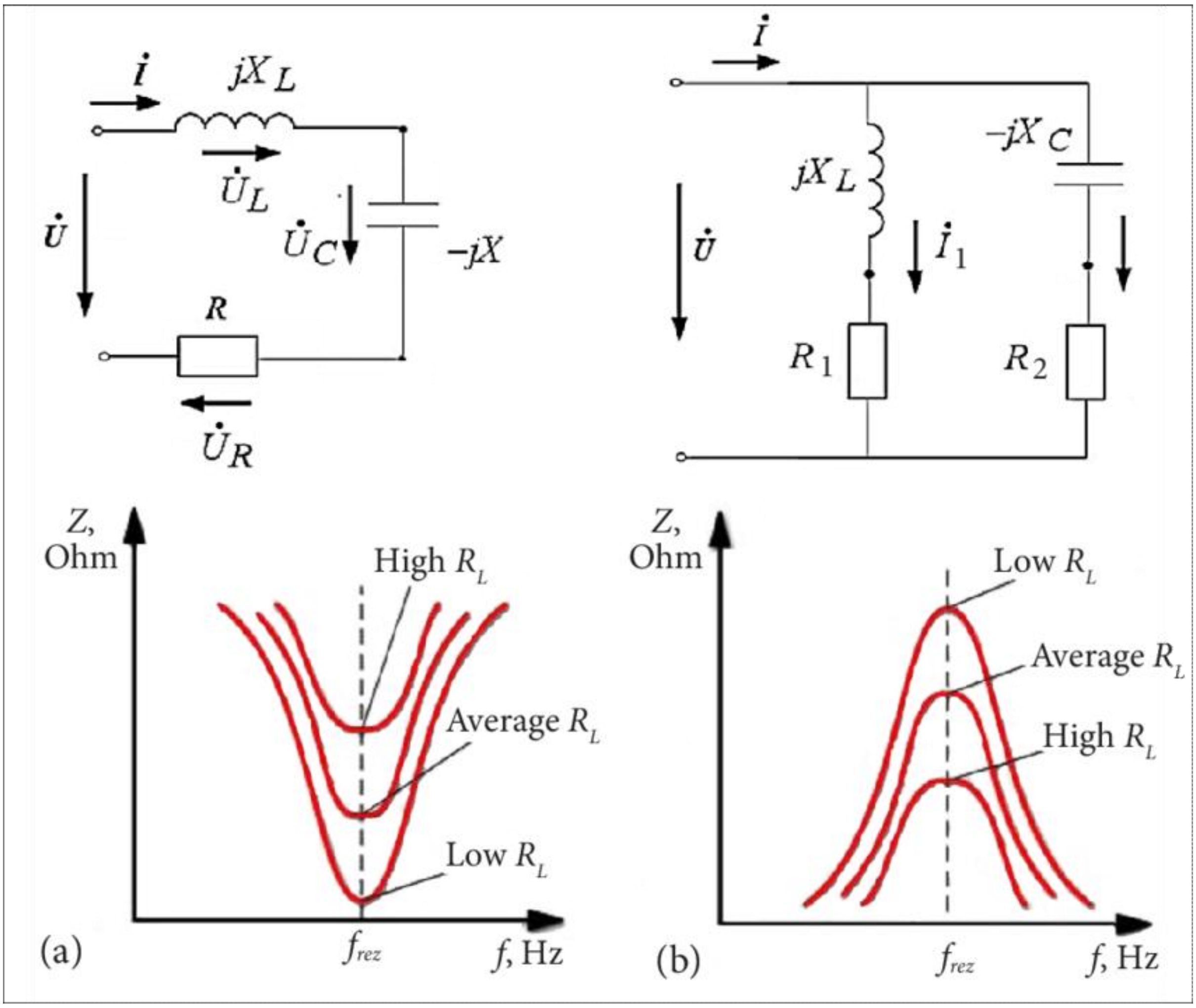


Figure 4. 11

(a) Net reactive current = 0

$$I_C - I_L = 0$$

Only the resistor branch draws current.

(b) Total line current is minimum

The source supplies only real current through resistor:

$$I_{\min} = \frac{V}{R}$$

(c) Line current and supply voltage are in phase

Power factor becomes **unity**.

(d) Impedance becomes maximum

For a parallel circuit, impedance at resonance is:

$$Z_{\max} = R$$

(Actually extremely high if the circuit has negligible losses.)

(e) Very large circulating currents in L and C

Although the supply current is minimum, the inductor and capacitor may exchange large reactive currents between themselves.

This is called **reactive current magnification**.

Comparison: Series vs Parallel Resonance

Feature	Series LCR	Parallel LCR
Current at resonance	Maximum	Minimum
Impedance	Minimum	Maximum
Behavior	Pure resistive	Pure resistive
Application	Filters, tuning circuits	Anti-resonance, stabilizers

Final Resonance Formula (Parallel)

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Q Factor

Introduction

The **Quality Factor**, commonly written as **Q factor**, is a dimensionless quantity that measures the **sharpness of resonance** of an AC circuit, especially in LCR circuits.

It tells us **how selective** or **efficient** a resonant circuit is in responding to a narrow band of frequencies.

A circuit with **high Q** has:

- sharp resonance peak
- low energy loss
- high voltage/current magnification

A circuit with **low Q** has:

- broad resonance
- higher energy loss
- less selectivity

Physical Meaning of Q Factor

The Q factor compares the **energy stored** in the circuit to the **energy dissipated** during one cycle.

$$Q = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipated per cycle}}$$

High Q means:

- circuit stores a lot of energy
- dissipates very little
- oscillations or current peak is sharp

Low Q means:

- circuit loses energy quickly

- resonance is broad

Q Factor of a Series LCR Circuit

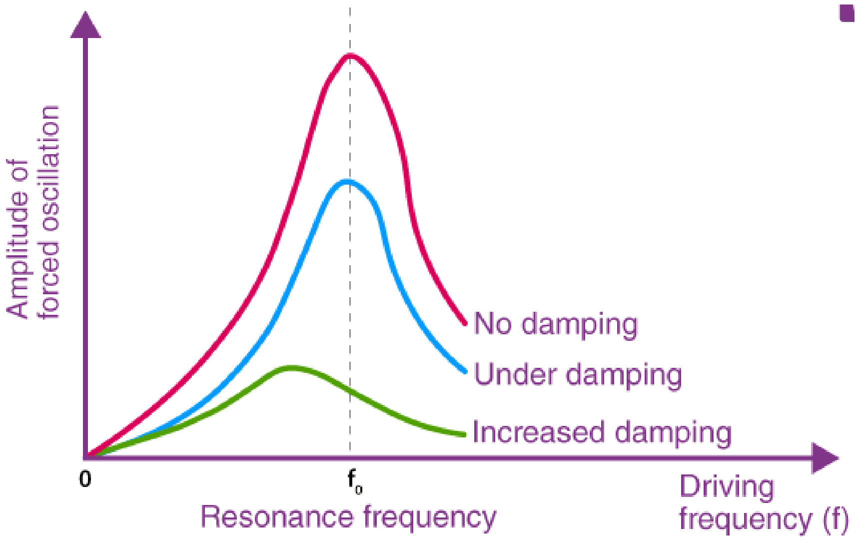


Figure 4. 12

In a series resonant circuit, the current reaches a **maximum** at resonance.

At resonance:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

The **Q factor** is:

$$Q = \frac{\omega_0 L}{R}$$

or equivalently,

$$Q = \frac{1}{\omega_0 CR}$$

Interpretation:

- Larger Lincreases Q
- Smaller Rincreases Q

- Circuit becomes “more selective”
- Current peak becomes sharper

Q Factor of a Parallel LCR Circuit

In a parallel resonant circuit, impedance becomes **maximum** at resonance and line current becomes **minimum**.

The Q factor is:

$$Q = R\sqrt{\frac{C}{L}}$$

Interpretation:

- Larger R increases Q
- Smaller L increases Q
- Circuit rejects unwanted frequencies more effectively

Q Factor and Bandwidth

One of the most important uses of Q factor is in communication systems like radio and TV tuning circuits.

The **bandwidth** (Δf) of the resonant peak is related to Q by:

$$Q = \frac{f_0}{\Delta f}$$

Where:

- f_0= resonant frequency
- $\Delta f = f_2 - f_1$= difference between half-power frequencies
- f_1 and f_2 are frequencies where power drops to **half** its maximum
- Current/voltage drops to $\frac{1}{\sqrt{2}}$ of peak value

Thus:

- **High Q → Small Bandwidth:** very selective
- **Low Q → Large Bandwidth:** less selective

Graphical Interpretation

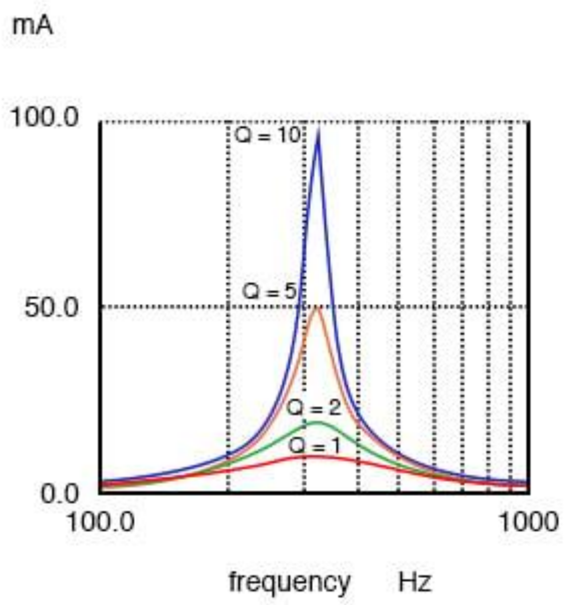


Figure 4. 13

- A **high Q** circuit shows a very **narrow and tall** resonance peak.
- A **low Q** circuit shows a **broad and short** peak.

Voltage and Current Magnification

In a **series** LCR circuit, at resonance:

$$Q = \frac{V_L}{V} = \frac{V_C}{V}$$

Meaning:

- Inductor voltage = Q times supply voltage
- Capacitor voltage = Q times supply voltage

This can lead to **very high voltages** across L and C (voltage magnification).

Importance of Q Factor

1. Determines the **selectivity** of radio receivers.
2. Helps design **filters** (band-pass, band-stop).
3. Determines the **efficiency** of oscillators.
4. Used in **power electronics** and **resonant converters**.
5. Indicates the **sharpness** of resonance.
6. Determines the **stability** of frequency-sensitive circuits.

Power factor

Introduction

In AC circuits, voltage and current are generally **not in phase** due to the presence of inductors and capacitors. Because of this phase difference, not all the supplied power is converted into useful (real) power.

The **Power Factor (PF)** is a measure of how effectively the electrical power is being used.

Definition of Power Factor

The **Power Factor** is defined as:

$$\text{Power Factor} = \cos\phi$$

where

ϕ= phase difference between **voltage** and **current**.

- If $\phi = 0^\circ$, voltage and current are in phase → PF = 1
- If $\phi$is large, the current lags or leads → PF decreases

Thus, PF indicates how efficiently the circuit converts electrical power into useful work.

AC Power Components

When an AC voltage V is applied to a circuit and current I flows, the **instantaneous power** varies with time due to phase difference.

Total power has three components:

(a) Real Power (P)

$$P = VI\cos\phi$$

Measured in **watts (W)**.

This is the useful power that actually performs work.

(b) Reactive Power (Q)

$$Q = VI\sin\phi$$

Measured in **VAR** (Volt-Ampere Reactive).

This power oscillates back and forth between the source and reactive components (L and C).

It does **no useful work**.

(c) Apparent Power (S)

$$S = VI$$

Measured in **VA**.

This is the total power supplied by the source.

These three are related by:

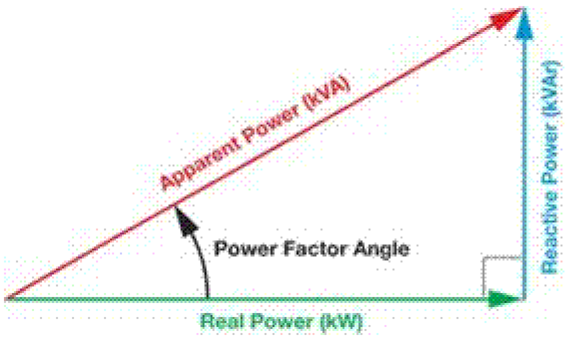


Figure 4. 14

$$S^2 = P^2 + Q^2$$

Power factor becomes:

$$\cos\phi = \frac{P}{S}$$

Power Factor in Different Types of Circuits

(a) Purely Resistive Circuit

- Voltage and current **in phase**
- $\phi = 0°$

$$\text{PF} = \cos 0 = 1$$

This is the **best** power factor.

(b) Purely Inductive Circuit

- Current **lags voltage by** $90°$
- $\phi = 90°$

$$\text{PF} = \cos 90 = 0$$

Circuit consumes **no real power**.

(c) Purely Capacitive Circuit

- Current **leads voltage by** $90°$
- $\mathrm{PF} = 0$

  Again, **no real power** is consumed.

(d) RL or RC Circuit (General Case)

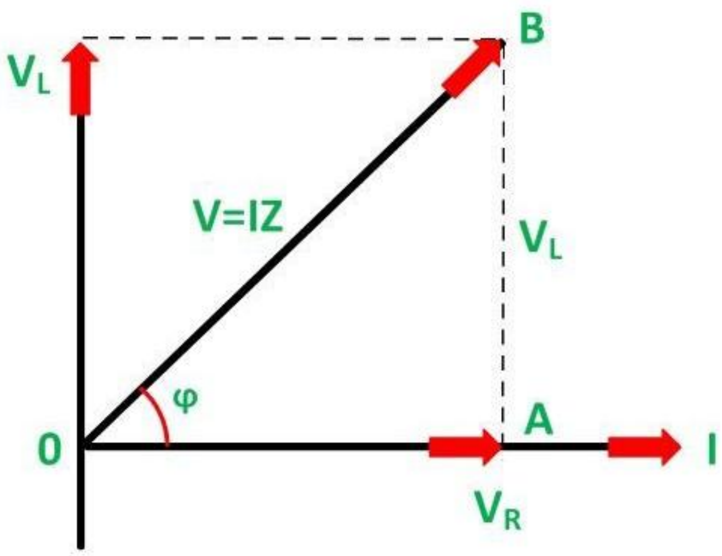


Figure 4. 15

Here:

- Inductive: current **lags** → lagging PF
- Capacitive: current **leads** → leading PF

The magnitude depends on the ratio:

$$\phi = \tan^{-1}(\frac{X_L - X_C}{R})$$

Importance of Power Factor

A low PF causes:

- Higher line current

- Increased heating in transformers and cables
- Excessive energy loss
- Voltage drops in power lines
- Bigger electricity bills (industries get penalized)

A high PF gives:

- Reduced current flow
- Lower power losses
- Better efficiency of electrical machines
- Lower electricity costs

Methods to Improve Power Factor

(a) Using Capacitors

Capacitors supply leading VARs and cancel lagging VARs from inductors.

Used in:

- capacitor banks
- ceiling fans
- industrial motors

(b) Using Synchronous Condensers

Overexcited synchronous motors improve the PF by supplying reactive power.

(c) Using Phase Advancers

Used in large induction motors to minimize lagging current.

Power Factor in LCR Series Circuit

At any frequency:

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

At resonance:

$$X_L = X_C$$

$$\cos\phi = 1$$

Thus, the series LCR circuit becomes purely resistive at resonance.

Unit 5: Maxwell's Equations and Electromagnetic Waves

1. Introduction
2. Maxwell's equations in vaccum, Material media
3. Physical Significance of Maxwell's Equations
4. Displacement current
5. Plane electromagnetic waves in free space
6. Velocity of light
7. Polynting Vector
8. Electromagnetic waves in a linear homogeneous media
9. Refractive index

Introduction

The study of electricity and magnetism took a major leap forward with the groundbreaking work of **James Clerk Maxwell**, who unified these two fields into a single, elegant framework known today as **classical electromagnetism**. Before Maxwell, electric and magnetic phenomena were treated as separate subjects. However, earlier discoveries — like Oersted's observation that electric currents produce magnetic fields and Faraday's law of electromagnetic induction — hinted that both were deeply connected.

Maxwell brought together all known laws of electricity and magnetism, added a crucial theoretical insight (the concept of **displacement current**), and formulated a complete set of four fundamental equations. These equations not only describe how electric and magnetic fields are generated and altered by charges, currents, and each other, but also predict something remarkable: **changing electric and magnetic fields can sustain each other and travel through space as waves**.

These self-propagating waves are known as **electromagnetic waves**, and they require **no physical medium** for transmission. Maxwell's theory showed that light itself is an electromagnetic wave — a revolutionary discovery that opened the door to understanding the entire electromagnetic spectrum, from radio waves to gamma rays.

Thus, Maxwell's Equations serve as the foundation of modern physics, forming the basis for technologies such as wireless communication, antennas, optics, radar, and more. Their introduction marked the beginning of the electromagnetic age and remains one of the greatest achievements in the history of science.

Maxwell's equations in vaccum, Material media

Maxwell's Equations form the foundation of classical electromagnetism. They describe how **electric fields (E)** and **magnetic fields (B)** behave, interact, and propagate. These equations exist in two useful forms:

1. **In Free Space (Vacuum)**
2. **In Material Media (Dielectrics, Conductors, Magnetic materials)**

The key difference between the two is the presence of **material response**:

- In vacuum → fields depend only on free charges and currents.
- In media → fields depend on **bound charges**, **polarization**, and **magnetization**.

Let's break them down clearly.

1. Maxwell's Equations in Vacuum

In vacuum, only **free charges** and **free currents** exist. The medium has fixed constants:

- Permittivity of free space: ε_0

- Permeability of free space: μ_0

The four equations are:

(1) Gauss's Law for Electricity

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

Meaning

- Electric charges create electric fields.
- The net electric flux outward from a closed surface equals the charge enclosed.

(2) Gauss's Law for Magnetism

$$\nabla \cdot \vec{B} = 0$$

Meaning

- There are **no magnetic monopoles**.
- Magnetic field lines are continuous loops with no beginning or end.

(3) Faraday's Law of Electromagnetic Induction

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Meaning

- A changing magnetic field induces an electric field.
- This induced field is non-conservative and forms closed loops.

(4) Ampère–Maxwell Law

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Meaning

- Magnetic fields are produced by
  - electric currents (J)
  - changing electric fields ($\frac{\partial E}{\partial t}$) → **displacement current**
- Maxwell's displacement current term is essential for predicting electromagnetic waves.

2. Maxwell's Equations in Material Media

When fields pass through a material, the atoms become polarized or magnetized.

Thus, we define:

- **Electric displacement field**:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

- **Magnetic field intensity**:

$$\vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M}$$

Where

- **P** = polarization (electric dipoles)
- **M** = magnetization (magnetic dipoles)

The equations rewrite to naturally incorporate bound charges and bound currents.

(1) Gauss's Law for Electricity in Material Media

$$\nabla \cdot \vec{D} = \rho_f$$

Meaning

- The flux of **D** depends only on **free charges**.

- Bound charges are absorbed inside **P**.

(2) Gauss's Law for Magnetism

$$\nabla \cdot \vec{B} = 0$$

This remains unchanged since **magnetic monopoles do not exist**.

(3) Faraday's Law

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Also unchanged — induction is a universal phenomenon.

(4) Ampère–Maxwell Law in Material Media

$$\nabla \times \vec{H} = \vec{J_f} + \frac{\partial \vec{D}}{\partial t}$$

- Magnetic fields now depend on
  - free currents (J^f)
  - changing electric displacement (D)

The response of materials is included indirectly through **D and H**.

3. Key Differences: Vacuum vs Material Media

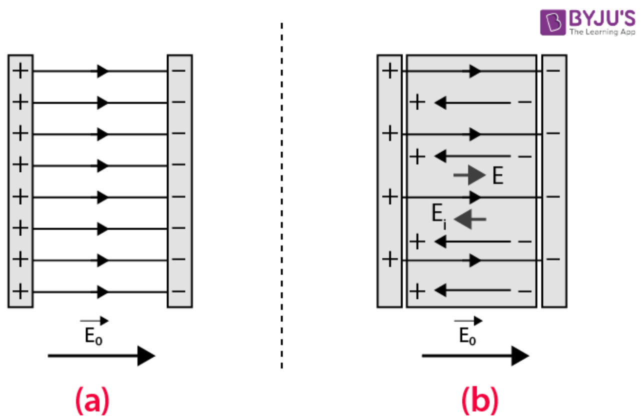


Figure 5. 1

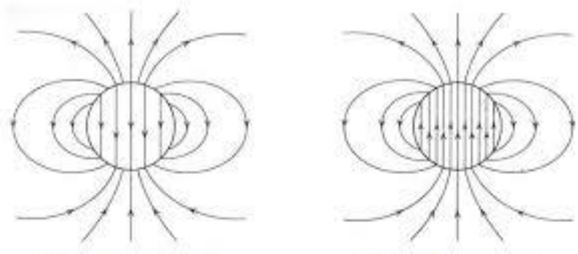


Figure 5. 2

Aspect	Vacuum	Material Media
Electric field relation	$D = \varepsilon_0 E$	$D = \varepsilon_0 E + P$
Magnetic field relation	$B = \mu_0 H$	$B = \mu_0(H + M)$
Free vs Bound	Only free charges/currents	Free + Bound charges/currents
Material constants	ε_0, μ_0	ε, μ, conductivity σ
Polarization, Magnetization	0	Present
Equations involve	E, B	D, E, H, B, P, M

Physical Significance of Maxwells's Equations

Maxwell's equations are not just mathematical statements; they capture the **fundamental laws of the electromagnetic universe**. Each equation reveals how electric and magnetic fields originate, interact, transform, and propagate through space. Together, they explain everything from the working of transformers to the behavior of light.

Let's go through the **physical meaning** of each equation one by one.

1. Gauss's Law for Electricity

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

Physical Significance

1. **Electric charges are the sources of electric fields.**

   If there is positive charge, field lines flow outward; if negative charge, field lines converge inward.

2. **Electric flux through a closed surface gives the total charge enclosed.**

   This tells us that electric fields spread out or converge only if charges are present.

3. **Fields cannot appear or disappear spontaneously.**

   They must originate from real physical charges.

This law defines the *origin* of electric fields and links electric phenomena directly with charge.

2. Gauss's Law for Magnetism

$$\nabla \cdot \vec{B} = 0$$

Physical Significance

1. **There are no magnetic monopoles.**

   Unlike electric charges, isolated north or south poles do not exist.

2. **Magnetic field lines are always closed loops.**

   They do not begin or end anywhere; they continue infinitely.

3. **Magnetism arises due to moving charges or intrinsic spin, not "magnetic charge."**

This law ensures that magnetic field lines are continuous, reinforcing the natural symmetry between electricity and magnetism.

3. Faraday's Law of Electromagnetic Induction

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Physical Significance

1. **A changing magnetic field produces an electric field.**

   This electric field is *induced* and forms closed loops.

2. **The induced electric field is non-conservative.**

   Unlike electrostatic fields, it does not originate from charges.

3. **This law explains generators, transformers, inductors, and electromagnetic induction.**

4. **This is one half of the reason electromagnetic waves exist.**

   A changing magnetic field induces an electric field → leading to self-sustaining wave formation.

This law is the foundation for electrical power generation and electromagnetic technologies.

4. Ampère–Maxwell Law

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Physical Significance

1. **Magnetic fields are produced by electric currents.**

   Current-carrying conductors generate circular magnetic fields.

2. **Changing electric fields also create magnetic fields.**

   This term — the displacement current — was added by Maxwell.

3. **This law completes the symmetry between electricity and magnetism.**

   Just as a changing magnetic field produces an electric field (Faraday's law),

   a changing electric field produces a magnetic field (Maxwell's correction).

4. **This is the second half of electromagnetic wave propagation.**

   One field creates the other, creating a continuously moving wave.

This law explains magnetism around wires, the functioning of capacitors in AC circuits, and the existence of EM waves.

5. Combined Physical Significance of All Four Equations

When taken together, Maxwell's equations express the following profound truths:

(i) Electromagnetic Fields Are Unified

Electric and magnetic fields are different aspects of a single physical entity:

The Electromagnetic Field.

Changes in one field create the other.

(ii) Electromagnetic Waves Exist

From Faraday's law and the Ampère–Maxwell law, we conclude:

- A changing **B** field induces **E**
- A changing **E** field induces **B**

Such mutual induction results in a *self-sustaining wave* that travels through space at the speed of light.

This leads directly to:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

So **light is an electromagnetic wave**.

(iii) They Explain All Electrical and Magnetic Phenomena

Everything in electromagnetism — from static charges to radio signals — fits inside these four laws.

- Electrostatics → Gauss's law
- Magnetostatics → Gauss's law for magnetism, Ampère's law
- Electromagnetic induction → Faraday's law
- AC circuits → Ampère–Maxwell law

- Optics → Maxwell's wave equations

(iv) Symmetry in Nature

Maxwell's correction introduced a beautiful symmetry:

- Changing **B** creates **E**
- Changing **E** creates **B**

This symmetry is the core idea behind unified field theories in modern physics.

(v) Conservation Laws

Maxwell's Equations automatically enforce:

- Conservation of charge
- Conservation of energy (via Poynting vector)
- Conservation of flux

These are fundamental laws of nature.

Displacement current

Introduction

In classical electromagnetism, electric current was traditionally understood as the flow of electric charges through a conductor. However, James Clerk Maxwell discovered that this definition was incomplete. There are situations where *magnetic fields are produced even when no actual charges are flowing*. To explain this, Maxwell introduced a new concept called **Displacement Current**.

This discovery unified electricity and magnetism and led directly to the prediction of **electromagnetic waves**, making displacement current one of the most important ideas in physics.

1. What is Displacement Current?

Displacement current is **not a real movement of charges**, but a term that arises due to a **changing electric field**.

It is defined as:

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

Where

- I_d= displacement current
- Φ_E= electric flux
- ε_0= permittivity of free space

Physical Meaning

- When the electric field changes with time, it produces an effect **exactly like a real current**.
- This "current" produces a magnetic field just as conduction current does.

Thus, displacement current ensures the **continuity of magnetic fields** in regions where physical charge flow is absent.

2. Why Did Maxwell Introduce Displacement Current?

Let’s consider a capacitor being charged.

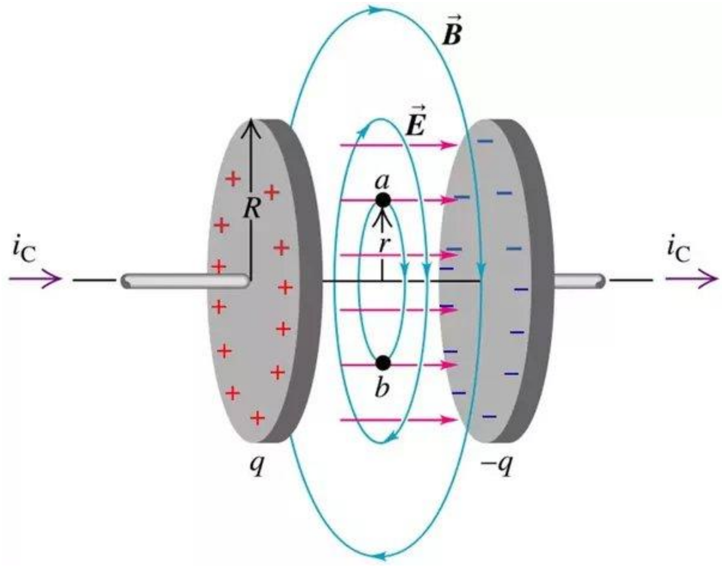


Figure 5. 3

- In the connecting wires → **conduction current** flows.
- Between capacitor plates → **no charge flows**, because it is an insulator or vacuum.
- Yet, experiments showed a magnetic field exists *between* the plates.

This was a contradiction in Ampère's law.

Ampère's Law (Before Maxwell)

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

This law works only where charge is flowing. But between capacitor plates, $J = 0$, so the law incorrectly predicts **no magnetic field**.

Maxwell solved this by adding a new current term.

Ampère–Maxwell Law

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Maxwell called

$$\varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

the **displacement current density** J_d.

This term allows a magnetic field to exist where there is **changing electric field**, even without real charge flow.

3. Mathematical Expression of Displacement Current

(i) Displacement Current Density

$$\overrightarrow{J_d} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Units: A/m²

This represents the "current density" produced by changing electric fields.

(ii) Total Displacement Current

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

Where $\Phi_E = \int \vec{E} \cdot d\vec{A}$is electric flux.

4. Physical Interpretation

(i) Exists Only When Electric Field is Time-Varying

A constant electric field produces no displacement current.

A changing electric field produces a displacement current.

(ii) It Creates Magnetic Fields

Just like conduction current, displacement current contributes to the magnetic field via the Ampère–Maxwell Law.

(iii) Ensures Continuity of Current

Without displacement current:

- Current entering one plate of a capacitor would have no matching current leaving the other plate.

- This violates the principle of charge conservation.

Displacement current “fills the gap” in non-conducting regions.

5. Displacement Current in a Capacitor

During charging or discharging:

- Conduction current flows in the wires.
- Displacement current flows **between** the plates.
- Both currents have the **same magnitude**, ensuring continuity.

Thus,

$$I_{\text{conduction}} = I_{\text{displacement}}$$

This keeps current continuous through the entire circuit.

6. Importance of Displacement Current

(i) Makes Ampere’s Law Consistent

With the displacement current term, Ampère's law is valid even in non-conducting regions.

(ii) Leads to Electromagnetic Waves

Displacement current enables a changing electric field to create a magnetic field, which in turn creates an electric field.

This self-sustaining process leads to wave propagation.

Maxwell derived the wave equation and obtained:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Thus, displacement current is fundamental for:

- light
- radio waves

- microwaves
- X-rays

(iii) Explains Behavior of Capacitors in AC Circuits

In AC systems:

- The electric field between capacitor plates changes continuously.
- Thus, displacement current flows, allowing AC to "pass through" capacitors even though they block DC.

(iv) Unifies Electricity and Magnetism

Because of displacement current, electricity and magnetism are unified into one theory — **electromagnetism**.

Plane electromagnetic waves in free space

Introduction

Electromagnetic (EM) waves are waves produced by the mutual, continuous variation of electric and magnetic fields. James Clerk Maxwell proved that a **time-varying electric field creates a magnetic field**, and a **time-varying magnetic field creates an electric field**. This continuous regeneration lets EM waves **travel through space without any medium** — even the vacuum. When these waves have their **electric field (E)** and **magnetic field (B)** distributed uniformly in planes perpendicular to the direction of propagation, we call them **plane electromagnetic waves**.

1. Transverse nature

- The electric field **E**, magnetic field **B**, and direction of wave propagation **k** are **mutually perpendicular**.
- This forms a **right-handed coordinate system**.

2. Propagation in Free Space

- Since there is **no medium**, the speed is:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m/s}$$

3. Uniform Plane Fronts

- At any instant, all points on a plane perpendicular to the direction of travel have **same field magnitude and direction**.
- This makes the wave a *plane* wave.

4. E and B are in Phase

Both fields reach their maxima and minima **at the same time**.

5. Energy Flow

- EM waves carry energy.
- The energy flow direction is given by the **Poynting vector**:

$$\vec{S} = \vec{E} \times \vec{B}$$

Mathematical Representation of Plane EM Waves

Consider a wave traveling along the **+x direction**.

Electric Field

$$\vec{E}(x,t) = E_0 \sin(kx - \omega t)\,\hat{j}$$

Magnetic Field

$$\vec{B}(x,t) = B_0 \sin(kx - \omega t)\,\hat{k}$$

Where:

- E_0, B_0: amplitudes
- $k = \frac{2\pi}{\lambda}$: wave number
- $\omega = 2\pi f$: angular frequency
- $kx - \omega t$: phase

Wave Impedance of Free Space

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377\ \Omega$$

This relates magnitudes of E and B:

$$E_0 = cB_0$$

Energy Density in EM Waves

- Electric energy density:

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

- Magnetic energy density:

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

In free space:

$$u_E = u_B$$

So total energy density:

$$u = \varepsilon_0 E^2$$

Physical Picture

- The **E field oscillates in one plane**.
- The **B field oscillates in a perpendicular plane**.
- Together, they march forward carrying energy.
- The fields are perfectly synchronized (in phase).
- The wave fronts are **infinite parallel planes** — thus "plane waves".

Velocity of light

Introduction

Maxwell's equations unified electricity and magnetism into a single framework. A key result from these equations is that a changing electric field produces a magnetic field, and a changing magnetic field produces an electric field. This mutual generation allows electromagnetic disturbances to propagate even in vacuum. When Maxwell calculated the speed of these waves, the value exactly matched the known speed of light. Thus, he concluded that **light is an electromagnetic wave**.

Maxwell's Equations in Free Space (Vacuum)

In free space, there is **no charge density** and **no current**.

Therefore, Maxwell's equations take simplified forms.

1. Gauss's Law for Electricity

$$\nabla \cdot \vec{E} = 0$$

2. Gauss's Law for Magnetism

$$\nabla \cdot \vec{B} = 0$$

3. Faraday's Law

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

4. Maxwell–Ampère Law

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Deriving the Velocity of Light

We use the curl equations since they connect variation of E and B fields in space and time.

Step 1: Curl of Faraday's Law

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}(\nabla \times \vec{B})$$

Step 2: Substitute Maxwell–Ampère Law

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Thus,

$$\nabla \times (\nabla \times \vec{E}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Step 3: Apply Vector Identity

Identity:

$$\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$$

In free space:

$$\nabla \cdot \vec{E} = 0$$

Thus,

$$-\nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Multiplying by –1:

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

This is a Wave Equation

The general wave equation is:

$$\nabla^2 \vec{E} = \frac{1}{v^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

Comparing with the above:

$$v^2 = \frac{1}{\mu_0 \varepsilon_0}$$

Therefore,

Velocity of Electromagnetic Waves in Free Space

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Using values of constants:

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

We get:

$$v = 3 \times 10^8 \text{ m/s}$$

This is exactly the **speed of light**.

Magnetic Field Wave Equation

Similarly, applying the same steps to Maxwell–Ampère law:

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

Thus, both E and B propagate with the same speed:

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Physical Interpretation

- A changing electric field induces a magnetic field.
- A changing magnetic field induces an electric field.
- These two fields continuously regenerate each other, allowing the disturbance to propagate through vacuum.
- The propagation speed is governed entirely by the electrical permittivity (ε_0) and magnetic permeability (μ_0) of free space.

Thus, the speed of light is not arbitrary; it is determined by the fundamental electromagnetic properties of vacuum.

Polynting Vector

Introduction

Electromagnetic waves carry energy as they propagate through space.

This energy flow is not random but directed, and its direction and magnitude are described by a quantity known as the **Poynting Vector**.

The concept was introduced by physicist **John Henry Poynting**, who showed that the cross-product of electric and magnetic fields represents the rate of energy transfer per unit area.

Definition of Poynting Vector

The **Poynting Vector** is defined as:

$$\vec{S} = \vec{E} \times \vec{B}$$

In SI units, the complete form uses permeability:

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$$

Meaning of the Poynting Vector

- **Direction:**

  The direction of **energy propagation** of the electromagnetic wave.

  It is perpendicular to both $\vec{E}$and $\vec{B}$(right-hand rule).

- **Magnitude:**

  The **rate of energy transfer per unit area**, i.e., **power per unit area**.

  Its unit is **Watt per square metre (W/m^2)**.

Simply put:

$$|\vec{S}| = \text{Energy flux}$$

Physical Interpretation

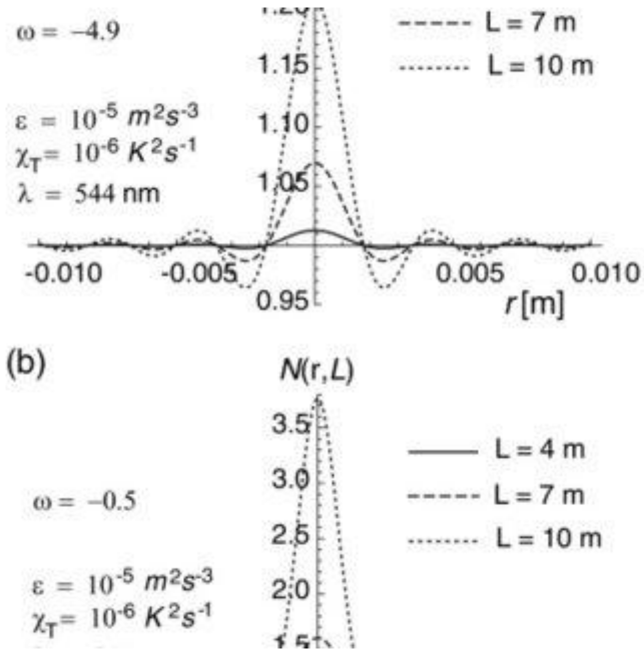


Figure 5. 4

In an electromagnetic wave:

- The **electric field oscillates** in one direction.
- The **magnetic field oscillates** in a direction perpendicular to the electric field.
- The **wave travels** in a direction perpendicular to both.

Thus:

$$\vec{S} \parallel \text{direction of EM wave propagation}$$

This shows that electromagnetic waves carry energy forward through space.

Instantaneous Poynting Vector

For a wave:

$$\vec{S}(t) = \frac{1}{\mu_0}(\vec{E}(t) \times \vec{B}(t))$$

But since $\vec{E}$and $\vec{B}$vary with time, $\vec{S}$also oscillates with time.

Average (Mean) Poynting Vector

For sinusoidal waves, we use the time averaged value:

$$\langle \vec{S} \rangle = \frac{1}{2\mu_0} E_0 B_0 \, \hat{n}$$

Using the relation $E_0 = cB_0$:

$$\langle \vec{S} \rangle = \frac{1}{2} \varepsilon_0 c E_0^2 \, \hat{n}$$

Where:

- E_0= peak electric field
- $\hat{n}$= direction of propagation

This averaged value is also called the **Intensity** of an EM wave.

Dimensional Analysis

$$[\vec{S}] = \frac{\text{Power}}{\text{Area}} = \frac{\text{W}}{\text{m}^2}$$

Energy Flow in EM Waves

- Energy in the electromagnetic wave is stored equally in both **electric** and **magnetic** fields.
- The Poynting vector quantifies how fast this energy moves through space.

Key Points to Remember

- **Poynting vector = direction and magnitude of EM energy flow.**
- It is given by:

$$\vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B})$$

- Direction is perpendicular to both E and B.

- Units: W/m^2
- Average value represents **intensity** of the wave.

Electromagnetic waves in a linear homogeneous media

Electromagnetic waves in a linear, homogeneous medium — detailed explanation

Definitions (what the words mean)

- **Linear**: the medium's constitutive relations are linear — fields scale linearly with sources. Specifically, **D = ε E** and **B = μ H** (and J = σ E for ohmic conduction). ε, μ, σ do not depend on field strength.
- **Homogeneous**: material parameters (ε, μ, σ) are the same at every point — no spatial variation.
- **Isotropic** (assumed unless stated): ε and μ are scalars (not tensors); response is same in every direction.

We consider time-harmonic fields with an $e^{-j\omega t}$convention (common in engineering). Real-time fields are Re{...} of phasors.

Maxwell's equations (frequency domain) in a source-free region

Using phasors (dependence $e^{-j\omega t}$) and assuming no impressed charge/current in the volume (sources can be at boundaries):

1. $\nabla \cdot \mathbf{D} = 0 \rightarrow \nabla \cdot (\varepsilon \mathbf{E}) = 0$
2. $\nabla \cdot \mathbf{B} = 0 \rightarrow \nabla \cdot (\mu \mathbf{H}) = 0$
3. $\nabla \times \mathbf{E} = -j\omega \mathbf{B} = -j\omega\mu \mathbf{H}$
4. $\nabla \times \mathbf{H} = \mathbf{J} + j\omega \mathbf{D} = (\sigma + j\omega\varepsilon)\mathbf{E}$

Define the complex permittivity

$$\tilde{\varepsilon} = \varepsilon' - j\varepsilon'' \text{with} \varepsilon'' = \frac{\sigma}{\omega}$$

so that $\sigma + j\omega\varepsilon = j\omega\tilde{\varepsilon}$.

Wave equation in a linear homogeneous medium

Take curl of Faraday's law and substitute Ampère's law:

$$\nabla \times (\nabla \times \mathbf{E}) = -j\omega\mu(\nabla \times \mathbf{H}) = -j\omega\mu(\sigma + j\omega\varepsilon)\mathbf{E}$$

Using $\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2\mathbf{E}$and $\nabla \cdot \mathbf{E} = 0$(for homogeneous, source-free), we get:

$$\nabla^2\mathbf{E} = \mu(\sigma + j\omega\varepsilon)j\omega\mathbf{E}$$

Rearrange to standard Helmholtz form:

$$\nabla^2\mathbf{E} = \gamma^2\mathbf{E}\text{where}\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$$

Similarly for **H**.

Here $\gamma$is the **complex propagation constant**:

$$\gamma = \alpha + j\beta$$

- α: attenuation constant (Np/m) — exponential decay
- β: phase constant (rad/m) — spatial phase progression

Closed-form for propagation constant and intrinsic impedance

Write γexplicitly:

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = j\omega\sqrt{\mu\varepsilon}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}$$

More usefully,

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

The **intrinsic impedance** of the medium (ratio of E to H) is:

$$\tilde{\eta} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{\mu}{\tilde{\varepsilon}}}\ \frac{1}{j}$$

(Use whichever algebraic form is convenient; note $\tilde{\eta}$is generally complex in a lossy medium.)

For a **lossless dielectric** ($\sigma = 0$):

$$\gamma = j\beta, \beta = \omega\sqrt{\mu\varepsilon}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\,\text{(real)}$$

Plane-wave solution (propagation in +z direction)

Assume a uniform plane wave with electric field polarized along $\hat{x}$, propagating in +z:

Phasor fields:

$$\mathbf{E}(z) = \hat{\mathbf{x}}\, E_0 e^{-\gamma z}$$

$$\mathbf{H}(z) = \hat{\mathbf{y}}\ \frac{E_0}{\tilde{\eta}} e^{-\gamma z}$$

In time domain (using $e^{-j\omega t}$convention):

$$\mathbf{E}(z,t) = \Re\{E_0 e^{-\gamma z} e^{-j\omega t}\}\, \hat{x}$$

If $\gamma = \alpha + j\beta$, then magnitude decays as $e^{-\alpha z}$and phase propagates with βz.

Important special cases and approximations

Lossless dielectric ($\sigma = 0$)

- $\gamma = j\beta, \beta = \omega\sqrt{\mu\varepsilon}$
- Wave speed $v = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}$

- Intrinsic impedance $\eta = \sqrt{\mu/\varepsilon}$(real)
- No attenuation: $\alpha = 0$

Weakly lossy dielectric ($\sigma \ll \omega\varepsilon$)

Expand:

$$\gamma \approx j\omega\sqrt{\mu\varepsilon}(1 - j\frac{\sigma}{2\omega\varepsilon}) = \alpha + j\beta$$

with

$$\alpha \approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}, \beta \approx \omega\sqrt{\mu\varepsilon}$$

Intrinsic impedance is $\tilde{\eta} \approx \sqrt{\frac{\mu}{\varepsilon}}(1 - j\frac{\sigma}{2\omega\varepsilon})$.

Good conductor ($\sigma \gg \omega\varepsilon$)

Approximate:

$$\gamma \approx (1 + j)\sqrt{\frac{\pi f\mu\sigma}{2}} = (1 + j)\sqrt{\frac{\omega\mu\sigma}{2}}$$

So $\alpha \approx \beta \approx \sqrt{\frac{\omega\mu\sigma}{2}}$.

Skin depth δ(distance where amplitude falls by $1/e$):

$$\delta = \frac{1}{\alpha} \approx \sqrt{\frac{2}{\omega\mu\sigma}}$$

Intrinsic impedance of a good conductor:

$$\tilde{\eta} \approx (1 - j)\sqrt{\frac{\omega\mu}{2\sigma}}$$

This is small and complex; fields penetrate only a small depth.

Energy, power flow and losses

Instantaneous Poynting vector (phasers)

Phasor Poynting vector (complex):

$$\tilde{\mathbf{S}} = \frac{1}{2}\mathbf{E} \times \mathbf{H}^*$$

Real part $\Re\{\tilde{S}\}$gives the **time-average power flow** (W/m²). For the plane wave above:

$$\langle \mathbf{S} \rangle = \frac{1}{2}\Re\{E_0 \frac{E_0^*}{\tilde{\eta}^*}\}\hat{z} = \frac{\mid E_0 \mid^2}{2}\Re\{\frac{1}{\tilde{\eta}}\}\hat{z}$$

Ohmic power loss (volume)

Power dissipated per unit volume due to conductivity:

$$P_{\text{loss, vol}} = \frac{1}{2}\sigma \mid \mathbf{E} \mid^2 \ (\text{W/m}^3)$$

In conductors or lossy dielectrics energy is absorbed and converted to heat.

Dispersion, phase velocity and group velocity

If ε(or μ) depends on frequency, medium is **dispersive**. Then:

- **Phase velocity**:

$$v_p = \frac{\omega}{\beta(\omega)}$$

- **Group velocity**:

$$v_g = \frac{d\omega}{d\beta(\omega)}$$

In nondispersive linear homogeneous media (ε, μ constant, real), $v_p = v_g = 1/\sqrt{\mu\varepsilon}$. In dispersive media they differ; pulse shapes change.

Boundary conditions (for interfaces between homogeneous media)

At the interface between two linear homogeneous media:

- Tangential components of **E**continuous:

$$\hat{n} \times (\mathbf{E}_1 - \mathbf{E}_2) = \mathbf{0}$$

- Tangential components of **H**continuous (unless surface current K_s):

$$\hat{n} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{K}_s$$

- Normal components of **D**jump by surface charge ρ_s:

$$\hat{n} \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s$$

- Normal components of **B**continuous:

$$\hat{n} \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0$$

From these come reflection/refraction (Fresnel) formulas for plane waves at interfaces — important for transmission lines, antennas, optics.

Polarization and wave types

- **Linear polarization**: **E**oscillates in one direction.
- **Circular/elliptical polarization**: two orthogonal components with phase difference.
- In unbounded homogeneous media, plane waves are transverse (E ⊥ k and B ⊥ k). In structures (waveguides), you can get TEM, TE, TM modes depending on boundary geometry.

Examples

1. **Lossless dielectric**:
   - $\beta = \omega\sqrt{\mu\varepsilon}$
   - $v = \frac{1}{\sqrt{\mu\varepsilon}}$
   - $\eta = \sqrt{\mu/\varepsilon}$
2. **Good conductor approximate**:
   - $\alpha \approx \beta \approx \sqrt{\frac{\pi f\mu\sigma}{2}}$
   - $\delta \approx \sqrt{\frac{2}{\omega\mu\sigma}}$
   - $\tilde{\eta} \approx (1-j)\sqrt{\frac{\omega\mu}{2\sigma}}$
3. **Loss tangent (measure of lossiness)**:
   - $\tan\delta = \frac{\sigma}{\omega\varepsilon}$(sometimes defined as $\varepsilon''/\varepsilon'$ in dielectrics)
   - If $\tan\delta \ll 1$→ low loss; if $\tan\delta \gg 1$→ conductor-like.

Physical intuition summary

- In a linear homogeneous medium, time-varying E and B sustain each other and propagate as waves whose spatial dependence is set by the complex propagation constant γ.
- The **real part** of γ(α) controls exponential **attenuation** (loss), the **imaginary part** (β) controls **phase progression**.
- Material parameters (ε, μ, σ) determine wave speed, impedance, attenuation — and therefore how energy moves and is dissipated.
- In perfect dielectrics, waves travel without loss at $1/\sqrt{\mu\varepsilon}$. In conductors, fields are confined to skin depth and dissipated as heat.

Refractive index

Refractive Index from Maxwell's Equations

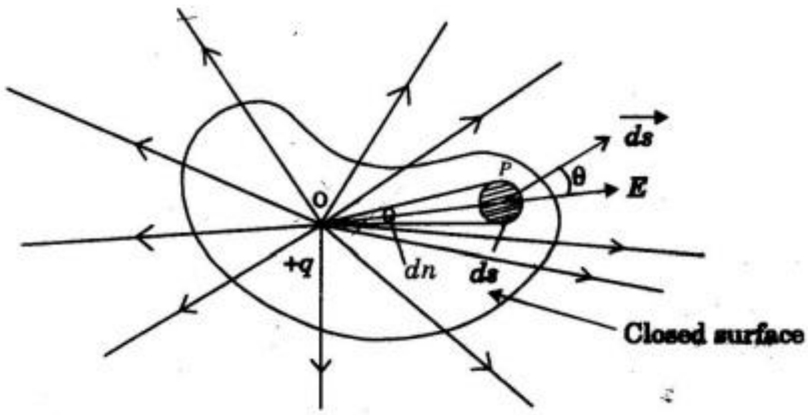


Fig. 2.1

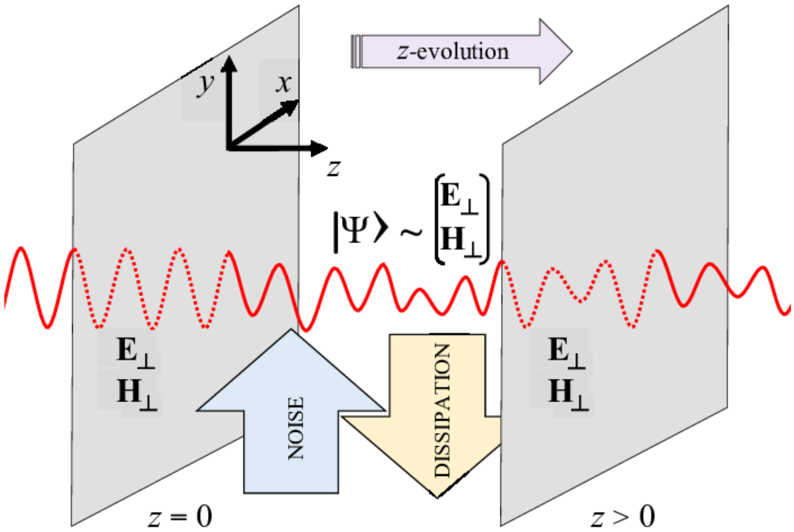


Figure 5. 5

When an electromagnetic (EM) wave travels through a material medium, its speed decreases due to the interaction of the fields with the material's permittivity and permeability. Maxwell's equations allow us to derive this speed and hence the **refractive index**.

1. Wave Equation in a Homogeneous Medium

In a non-conducting ($\sigma = 0$), linear, homogeneous, isotropic medium:

Maxwell's curl equations are:

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t}$$

Taking the curl of the first equation:

$$\nabla \times (\nabla \times \mathbf{E}) = -\mu \frac{\partial}{\partial t}(\nabla \times \mathbf{H})$$

Substitute the second curl equation:

$$\nabla \times (\nabla \times \mathbf{E}) = -\mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

Using the vector identity:

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$$

In a charge-free medium:

$$\nabla \cdot \mathbf{E} = 0$$

Hence:

$$\nabla^2 \mathbf{E} = \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

This is the **wave equation in a medium**.

2. Velocity of EM Wave in the Medium

From the wave equation:

$$\text{wave speed } v = \frac{1}{\sqrt{\mu\varepsilon}}$$

In vacuum:

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

3. Derivation of Refractive Index

The refractive index is defined as:

$$n = \frac{c}{v}$$

Substituting the expressions for $c$and v:

$$n = \frac{1/\sqrt{\mu_0\varepsilon_0}}{1/\sqrt{\mu\varepsilon}}$$

$$n = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$$

For most non-magnetic materials:

$$\mu \approx \mu_0$$

Thus:

$$n = \sqrt{\frac{\varepsilon}{\varepsilon_0}}$$

Let relative permittivity be:

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

So:

$$n = \sqrt{\varepsilon_r}$$

This is the **refractive index obtained directly from Maxwell's equations**.

4. Physical Meaning

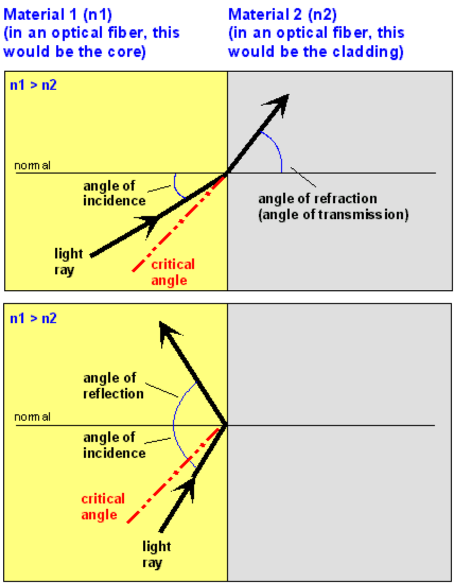


Figure 5. 6

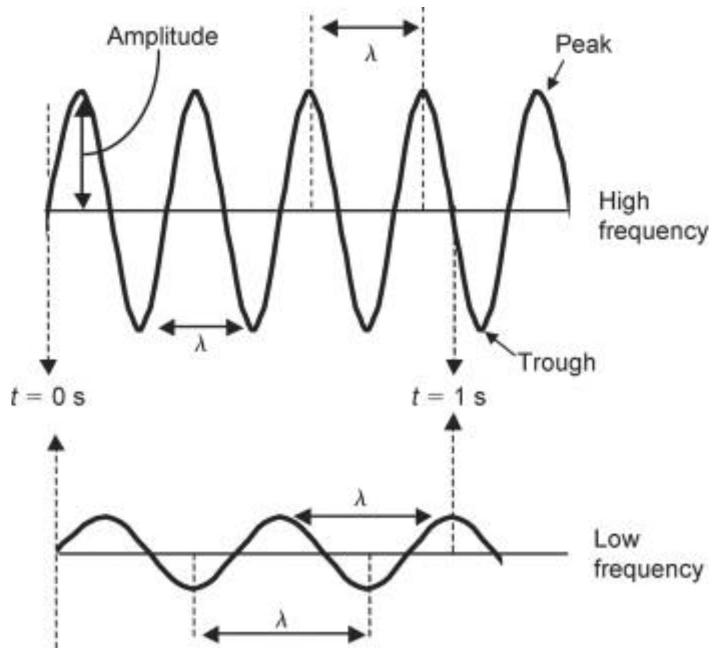


Figure 5. 7

• A medium with larger permittivity slows EM waves more.

• Therefore the refractive index increases.

• The bending of light at boundaries arises because wave speed changes according to Maxwell's equations.